Available online at www.refaad.com
Gen. Lett. Math., 9(1) (2020), 42- 45
Research Article

General Letters in Mathematics (GLM)

Journal Homepage: http://www.refaad.com/views/GLM/home.aspx

ISSN: 2519-9277 (Online) 2519-9269 (Print)

DHO conjugate gradient method for unconstrained optimization

Dlovan Haji Omar [a*]

[a] Department of Mathematics, Faculty of Science, University of Zakho, Zakho, Kurdistan Region, Iraq.

Abstract

In this paper, we suggest a new coefficient conjugate gradient method for nonlinear unconstrained optimization by using two parameters one of them is parameter of (FR) and the other one is parameter of (CD), we give a descent condition of the suggested method.

Keywords: Conjugate Gradient Method; Descent Condition; Optimization problems.
2010 MSC No: 74Pxx, 78M50, 90C26.

1. Introduction

Consider the unconstrained optimization problem:

$$\text{Min } f(x), x \in R^n \tag{1.1}$$

where $f: R^n \to R$ is a real-valued, continuously differentiable function.
A nonlinear conjugate gradient method generates a sequence $\{x_k\}$, k ≥ 1, starting from an initial guess $x_1 \in R^n$, using the recurrence

$$x_{k+1} = x_k + \alpha_k d_k \tag{1.2}$$

Or

$$v_k = \alpha_k d_k$$

Where, $v_k = x_{k+1} - x_k$

The positive step size α_k is obtained by some line search, and d_k is a search direction. Normally the search direction at the first iteration is the steepest descent direction, namely $d_1 = -g_1$ and the other search directions can be defined as:

$$d_{k+1} = -g_{k+1} + \beta_k d_k \tag{1.3}$$

Where $g_k = \nabla f(x_k)$ and β_k is a scalar. There are many formulas for β_k, for example, Hestenes-Stiefel (HS) [5] , Fletcher-Reeves (FR) [6], Polak-Ribiere-Polyak (PRP) [2] , Dai and Yuan (DY) [10], (CD) [7, Dlovan H. O. [3], Shareef, S.G , Khatab, H.A. and Ismael, S.S.[9] and Shareef, S.G , Dlovan H. O.[8], these formulas are as follows:

$$\beta_k^{HS} = \frac{g_{k+1}^T(g_{k+1}-g_k)}{d_k^T(g_{k+1}-g_k)} \tag{1.4}$$

$$\beta_k^{FR} = \frac{\|g_{k+1}\|^2}{\|g_k\|^2} \tag{1.5}$$

$$\beta_k^{PRP} = \frac{g_{k+1}^T(g_{k+1}-g_k)}{\|g_k\|^2} \tag{1.6}$$

* Corresponding author
Email address: dilovan.omar@uoz.edu.krd (Dlovan Haji Omar)
doi: 10.31559/glm2020.9.1.5
Received: 22 Dec 2019: Accepted 14 Jul 2020

$$\beta_k^{DY} = \frac{\|g_{k+1}\|^2}{d_k^T(g_{k+1}-g_k)} \tag{1.7}$$

$$\beta_k^{CD} = \frac{\|g_{k+1}\|^2}{-d_k^T g_k} \tag{1.8}$$

$$\beta_k^{DH} = \mu \frac{\|g_{k+1}\|^2}{-d_k^T g_k}(1 + (k\frac{d_k^T g_{k+1}}{\|y_k\|^2})(\frac{\|g_{k+1}\|^2}{d_k^T g_k})) \tag{1.9}$$

$$\beta_k^{SHS} = \mu \frac{\|g_{k+1}\|^2}{d_k^T g_k}(1 - \frac{g_{k+1}^T v_k}{d_k^T y_k}\frac{\|g_{k+1}\|^2}{d_k^T y_k}) \tag{1.10}$$

$$\beta_k^{SD} = (1-\theta_k)\frac{g_{k+1}^T y_k}{d_k^T y_k} + \theta_k[\mu \frac{g_{k+1}^T y_k}{d_k^T y_k}(1 - 2\frac{d_k^T g_{k+1}}{\|y_k\|^2}(\frac{g_{k+1}^T y_k}{d_k^T y_k})) \tag{1.11}$$

$y_k = g_{k+1} - g_k$, symbol $\|.\|$ denotes the Euclidean norm of vectors. The most well studied properties of conjugate gradient methods are its global convergence properties. The convergence of conjugate gradient methods under different line searches has been studied by many authors, such as Gilbert and Nocedal [4], and Hestenes and Stiefel [5].

This paper is organized as follow: in Section 2, we will suggest a new conjugate gradient method. In Section 3, we prove the descent condition of new method. In Section 4, some numerical experiments of the new conjugate gradient method. In section 5, we will give the conclusion.

2. New conjugate gradient algorithm

In this section, we will derive the our suggestion
Consider the complex number

$$z = x + iy \tag{2.1}$$

Suppose that

$$z = \beta^{FR} + i\beta^{CD} \tag{2.2}$$

Then,

$$\beta_k^{New} = |z| = \sqrt{(\beta^{FR})^2 + (\beta^{CD})^2}$$

Implies that

$$\beta_k^{New} = \sqrt{(\frac{\|g_{k+1}\|^2}{\|g_k\|^2})^2 + (\frac{\|g_{k+1}\|^2}{-d_k^T g_k})^2} \tag{2.3}$$

And since $d_k = -g_k$, so

$$\beta_k^{New} = \sqrt{(\frac{\|g_{k+1}\|^2}{\|g_k\|^2})^2 + (\frac{\|g_{k+1}\|^2}{\|g_k\|^2})^2}$$

Implies that

$$\beta_k^{New} = \sqrt{2(\frac{\|g_{k+1}\|^2}{\|g_k\|^2})^2}$$

Or

$$\beta_k^{New} = \sqrt{2}\frac{\|g_{k+1}\|^2}{\|g_k\|^2}$$

and let

$$\beta_k^{New} = \delta \frac{\|g_{k+1}\|^2}{\|g_k\|^2} \tag{2.4}$$

Where $\delta = \sqrt{2}\gamma$. $\gamma > 0$

2.1 Algorithm of a new conjugate gradient method(β_k^{NEW})

Step (1): Select x_1 and $\varepsilon = 10^{-5}$.
Step (2): Set $d_1 = -g_1$, $g_k = \nabla f(x_k)$, Set $k = 1$.
Step (3): Compute the step length $\alpha_k > 0$ satisfying the Wolfe line search

$$f(x_k + \alpha_k d_k) - f(x_k) \le c_1 \alpha_k g_k^T d_k$$

$$|g_{K+1}^T d_k| \le c_2 |g_k^T d_k|$$

where, $0 < c_1 < c_2 < 1$.
Step (4):- Compute

$$x_{k+1} = x_k + \alpha_k d_k .$$

$$g_{k+1} = \nabla f(x_{k+1}), If \; \|g_{k+1}\| \le \varepsilon, \text{then stop.}$$

Step (5):- Compute β_k^{NEW} by (2.4)

Step (6): Compute $d_{k+1} = -g_{k+1} + \beta_k^{NEW} d_k$

Step (7): If $|g_{k+1}^T g_k| > 0.2\|g_{k+1}\|^2$ then go to step 2.

Else

$k = k + 1$ and go to step 3.

Theorem 2.1: Assume that the sequence $\{x_k\}$ is generated by (1.2), then the search direction in (1.3) with new conjugate gradient method (2.4) satisfy the descent condition, i.e. $d_{k+1}^T g_{k+1} \leq 0$ with exact and inexact line search.

Proof:

The proof is done induction, the result clearly holds for $k = 0$

$$d_0^T g_0 = -\|g_0\|^2 \leq 0$$

Now, we prove the current search direction in descent direction at the (k+1)

From (1.3) and (2.5) we have,

$$d_{k+1} = -g_{k+1} + \beta_k^{New} d_k \tag{2.5}$$

$$d_{k+1}^T g_{k+1} = -\|g_{k+1}\|^2 + \delta \frac{\|g_{k+1}\|^2}{\|g_k\|^2} d_k^T g_{k+1} \tag{2.6}$$

If the step length $\alpha_k$is chosen by an exact line search which requires $d_k^T g_{k+1} = 0$. Then the proof is complete. If the step length $\alpha_k$is chosen by inexact line search which requires $d_k^T g_{k+1} \neq 0$,

Since the parameter of (FR) is satisfies the descent condition then the above equation is satisfy the descent condition.

3. Numerical Results

This section is devoted to test the implementation of new method. We compare the our method with Conjugate Gradient (FR and PR), the comparative tests involve Well-known nonlinear problems (standard test function) with different dimensions$4 \leq n \leq 5000$, all programs are written in FORTRAN90 language and for all cases the stopping condition is $\|g_{k+1}\| \leq 10^{-5}$ the results given in table (1) specifically quote the number of function NOF and the nuber of iteration NOI . More experimental results in table (1) confirm that the new CG is superior to standard CG method with respect to the NOI and NOF.

Table (1): Comparative Performance of the Two Algorithms (FR, PR and New Conjugate Gradient Method)

Test function	N	Algorithm of FR		Algorithm of PR		New algorithm	
		NOI	NOF	NOI	NOF	NOI	NOI
Rosen	4	30	85	30	83	30	30
	100	30	85	30	83	30	30
	500	30	85	30	83	30	30
	1000	30	85	30	83	30	30
	5000	30	85	30	83	30	30
Wood	4	26	60	29	67	26	60
	100	27	62	30	69	26	60
	500	27	62	30	69	26	60
	1000	27	62	30	69	26	60
	5000	27	62	30	69	26	60
Wolf	4	11	23	11	24	11	23
	100	45	91	49	99	45	90
	500	46	93	52	105	45	90
	1000	52	105	70	141	52	104
	5000	144	293	165	348	141	293
Central	4	18	123	22	159	12	49
	100	24	194	22	159	13	61
	500	28	251	23	171	22	176
	1000	28	251	23	171	23	188
	5000	28	251	30	270	23	188
Cubic	4	13	38	15	45	13	38
	100	14	40	16	47	13	38
	500	15	44	16	47	13	38
	1000	15	44	16	47	13	38
	5000	15	44	16	47	13	38
Powell (3)	4	F	F	40	109	14	33
	100	F	F	42	123	22	49
	500	F	F	43	125	25	55
	1000	F	F	43	125	26	57
	5000	F	F	43	125	26	57
Powell (4)	4	40	100	40	120	39	106
	100	42	123	43	135	39	106
	500	43	125	46	150	39	106
	1000	43	125	46	150	39	106
	5000	43	125	50	180	39	106
Total		1413	4430	1281	3980	1040	2683

Table (2): Percentage of Improving of the New Method

	Algorithm of FR& PR %	New Algorithm with FR	New Algorithm with PR
NOI	100%	73.60226%	81.18657%
NOF	100%	60.56433%	67.41206%

4. Conclusion

In this paper, we suggested a new conjugate gradient method for unconstrained optimization. Implemented and tested to some extent, while numerical tests were carried out, on low and high dimensionality problems, and comparisons were made amongst different test functions with inexact line search. Some of the numerical results have been reported. In future we can use the new conjugate gradient method with other standard conjugate method to obtain the three terms conjugate gradient method [1].

References

[1] Y.H. Dai and Y. Yuan, *A nonlinear conjugate gradient with strong global convergence properties*, SIAM J. Optim. 10(1999), 177–182.

[2] H. Dlovan, *A new Suggested Conjugate Gradient Algorithm with Logistic Mapping*, Journal of University of Zakho, 4(2)(2016), 244-247, https://doi.org/10.25271/2016.4.2.113.

[3] R. Fletcher and C. Reeves, *Function minimization by conjugate gradients*, Comput. J., 7(2) (1964), 149–154, https://doi.org/10.1093/comjnl/7.2.149.

[4] R., Fletcher, Practical Methods of Optimization vol.1: Unconstrained Optimization. Jhon Wiley & Sons, New York, 1987.

[5] J. C. Gilbert and J. Nocedal, *Global convergence properties of conjugate gradient methods for optimization*, SIAM J. Optim. 2 (1992), 21-42, https://doi.org/10.1137/0802003.

[6] M.R. Hestenes and E. Steifel, *Method of conjugate gradient for solving linear equations*, J.Res. Nat. Bur. Stand. 49(1952), 409–436.

[7] L. Ibrahim, and S. G. Shareef, *A new class of three-term conjugate gradient methods for solving unconstrained minimization problems*, General Letters in Mathematics, 7(2)(2019), 79-86, https://doi.org/10.31559/glm2019.7.2.4.

[8] E. Polak, and G. Ribiere, *note sur la convergence de directions conjugees*, Rev. Francaise Inform. Recherche Operationelle 3(1969), 35–43, https://doi.org/10.1051/m2an/196903r100351.

[9] S.G. Shareef and H. Dlovan, *New Proposed Conjugate Gradient Method for Nonlinear Unconstrained Optimization*, Journal of University of Zakho, 4(2) (2016), 248-252, https://doi.org/10.25271/2016.4.2.114.

[10] S.G. Shareef, H.A. Khatab and S.S. Ismael, *New conjugate gradient method for unconstrained optimization with logistic mapping*, Jornal University of Zakho , 4(1)(2016), 133-136, https://doi.org/10.25271/2016.4.1.32.