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Comment from article "Two different scenarios when the Collatz Conjecture fails"

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Abstract

This work substantiates the essence of the erroneous conclusion of the author of the work 'M. Ahmed, Two different scenarios when the Collatz Conjecture fails. General Letters in Mathematics. 2021' about the false of Collatz's hypothesis.

Keywords: Recurrence sequence, Jacobsthal numbers, Collatz conjecture.
2010 MSC: 37P99, 11Y16, 11A51, 11-xx, 11Y50

1. Introduction

'The Collatz Conjecture is false. End of story', this is how the work ends [1]. This is a bold conclusion and could not be ignored. This work shows that the conclusion made is hasty and here is why.
Consider sequences of numbers $\{\Im_{\theta,n}\}_{n=0}^{\infty}$, parametrized by an odd natural number θ whose terms are determined by the equality

$$\Im_{\theta,n} = \frac{1}{3}[\theta \cdot 2^n - 1], \quad n \quad \in \quad N \quad \cup \quad \{0\} \tag{1.1}$$

and group them by even and odd powers as

$$3 \cdot \Im_{\theta,n} \to \begin{cases} \quad \theta \cdot 2^1 - 1 \quad\quad \theta \cdot 2^3 - 1 \quad\quad \theta \cdot 2^5 - 1 \quad\quad \theta \cdot 2^7 - 1 \quad\quad \dots \\ \theta \cdot 2^0 - 1 \quad\quad \theta \cdot 2^2 - 1 \quad\quad \theta \cdot 2^4 - 1 \quad\quad \theta \cdot 2^6 - 1 \quad\quad\quad \dots \end{cases} \tag{1.2}$$

as:

$$\begin{cases} p_{\theta,l} = \frac{1}{3}[\theta \cdot 2^l - 1], \quad l = 1, 3, 5, 7 \dots \\ m_{\theta,k} = \frac{1}{3}[\theta \cdot 2^k - 1], \quad k = 0, 2, 4, 6 \dots \end{cases} \tag{1.3}$$

where θ is the index of sequences of numbers $\theta \cdot 2^n$.

Let's divide the subset of odd natural numbers into three groups, as shown in Fig.1, and formulate the following properties of numbers (1.3).

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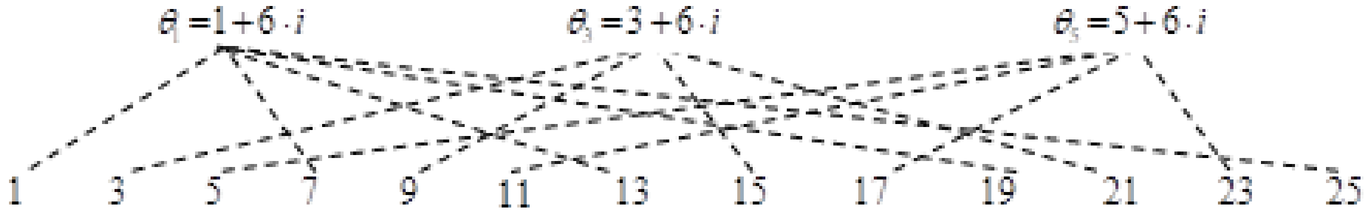

Figure 1:

Property 1.For multiples of three values

$$\theta = 1 + 6 \cdot i, \quad i = 0, 1, 2, 3, 4, 5, \ldots \tag{1.4}$$

and

$$\theta = 5 + 6 \cdot i, \quad i = 0, 1, 2, 3, 4, 5, \ldots \tag{1.5}$$

numbers (1.3) are equals

$$\begin{cases} p_{\theta,l} = \text{noninteger}, \\ m_{\theta,k} = \text{integer}. \end{cases} \quad \textbf{and} \quad \begin{cases} p_{\theta,l} = \text{integer}, \\ m_{\theta,k} = \text{noninteger}. \end{cases} \tag{1.6}$$

in accordance. For multiples of three values

$$\theta = 3 + 6 \cdot i, \quad i = 0, 1, 2, 3, 4, 5, \ldots \tag{1.7}$$

numbers (1.3) are equals

$$\begin{cases} p_{\theta,l} = \text{noninteger}, \\ m_{\theta,k} = \text{noninteger}. \end{cases} \tag{1.8}$$

Property 2. For each value of θ, sequences of integers (1.3) are solutions of the second-order recurrent equation

$$m(p)_{\theta,k(l)+2} = 4m(p)_{\theta,k(l)} + 1 \tag{1.9}$$

and initial conditions

$$m(p)_{\theta,2(3)} - m(p)_{\theta,0(1)} = 1(2) \cdot \theta \tag{1.10}$$

Let's confirm properties 1 and 2 with concrete calculations:

Shaded cells in Table 1 correspond to fractional values of numbers (1.3)

So, if the index θ changes as $\theta_{1,i} = 1 + 6 \cdot i$ fig.1, and in calculations Table 1 $\theta_1 = 1, 7$, then the numbers $m_{\theta,k}$ take integer values, and the numbers $p_{\theta,l}$ take fractional values. If the index θ changes as $\theta_{5,i} = 5 + 6 \cdot i$ (fig.1), and in calculations Table 1 $\theta_5 = 5, 11$, then the numbers $m_{\theta,k}$ take fractional values, and the numbers $p_{\theta,l}$ take integer values. If the index θ is a multiple of three (in Fig. 1), the series $(\theta_3 = 3 + 6 \cdot i)$ and in calculations Table 1 $\theta_3 = 3$, then both numbers $m_{\theta,k}$, $p_{\theta,l}$ are fractional. Calculations Table 1 also confirm rule (1.9).

From (1.3) and properties 1 and 2, we formulate a system of equations:

$$\begin{cases} \theta = 1 + 6 \cdot i, & (a) \\ m_{\theta,k} = \frac{1}{3}[\theta \cdot 2^k - 1] = \text{integer}, & (b) \\ 3m_{\theta,k} + 1 = \theta \cdot 2^k. & (c) \end{cases} \quad \begin{cases} \theta = 5 + 6 \cdot i, & (d) \\ p_{\theta,l} = \frac{1}{3}[\theta \cdot 2^l - 1] = \text{integer}, & (e) \\ 3p_{\theta,l} + 1 = \theta \cdot 2^l. & (f) \end{cases} \tag{1.11}$$

Table 1:

		1		**3**		**5**		**7**		**9**		l
	0		**2**		**4**		**6**		**8**		**10**	k
$\theta = 1$	0		1		5		21		85		341	$m_{\theta,k}$
												$p_{\theta,l}$
$\theta = 3$												$m_{\theta,k}$
												$p_{\theta,l}$
$\theta = 5$												$m_{\theta,k}$
		3		13		53		213		853		$p_{\theta,l}$
$\theta = 7$	2		9		37		149		597		2389	$m_{\theta,k}$
												$p_{\theta,l}$
$\theta = 11$												$m_{\theta,k}$
		7		29		117		469		1877		$p_{\theta,l}$

According to (1.11), every odd number θ generates sequences $\theta \cdot 2^n$ (1.12)

$$\begin{cases} \theta \cdot 2^1 = 3p_{\theta,0} + 1, \quad \theta \cdot 2^3 = 3p_{\theta,2} + 1 \quad \theta \cdot 2^5 = 3p_{\theta,4} + 1 \quad \theta \cdot 2^7 = 3p_{\theta,6} + 1 & (a) \\ \quad\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow & \\ \theta \cdot 2^0, \quad \theta \cdot 2^1, \quad \theta \cdot 2^2, \quad \theta \cdot 2^3, \quad \theta \cdot 2^4, \quad \theta \cdot 2^5, \quad \theta \cdot 2^6, \quad \theta \cdot 2^7 & (b) \\ \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow & \\ \theta \cdot 2^0 = 3m_{\theta,0} + 1, \quad \theta \cdot 2^2 = 3m_{\theta,2} + 1 \quad \theta \cdot 2^4 = 3m_{\theta,4} + 1 \quad \theta \cdot 2^6 = 3m_{\theta,6} + 1 & (c) \end{cases} \tag{1.12}$$

Construction (1.12) is an element of a kind of Lego, in which integers (1.11b,e) generate sequences with corresponding values. The Lego made from (1.11) is shown in Fig. 2. Here, arrows $\leftrightarrow, \updownarrow$ show transitions $\frac{1 \cdot 2^4 - 1}{3} = 5 \cdot 2^0$, $\frac{13 \cdot 2^2 - 1}{3} = 17 \cdot 2^0$, $\frac{17 \cdot 2^1 - 1}{3} = 11 \cdot 2^0$, $\frac{11 \cdot 2^1 - 1}{3} = 7 \cdot 2^0$, $\frac{7 \cdot 2^6 - 1}{3} = 37 \cdot 2^0$.

Arrows $\Rightarrow\Rightarrow\Rightarrow$ show transitions according to rule (1.9). Values of numbers in square brackets $[\theta]$ are multiples of three.

Arrows $-\blacktriangleright$ show the beginning of the trajectory of the sequence of Example 3. [1]. This is a periodically growing oscillating sequence. It is built on the basis of the reverse algorithm (Lemma 3.1). However, sequences with multiples of three indices do not generate other sequences Table 1, which forced the author to postulate the equivalence of sequences of the type $[\theta]$ to sequences with indices $4[\theta] + 1$ and to apply rule (1.9) to implement transitions between them.

But the assumption of equivalence between the sequences $[\theta]$ and $4[\theta] + 1$ does not agree with the regularities in Fig.1 and Fig.2 and the results of calculations Table 1. Unlike sequences $\theta_1 = 1 + 6 \cdot i$ and $\theta_5 = 5 + 6 \cdot i$ integers of type (1.3), sequences $\theta_3 = 3 + 6 \cdot i$ of numbers with properties 1 and 2 do not generate.

Conclusion

The periodically oscillating increasing sequence of Example 3.7 [1] is not related to the well-known Collatz problem [2], and does not indicate failure of the Collatz hypothesis. So, the book "Collatz Hypothesis" is not closed, and the story continues.

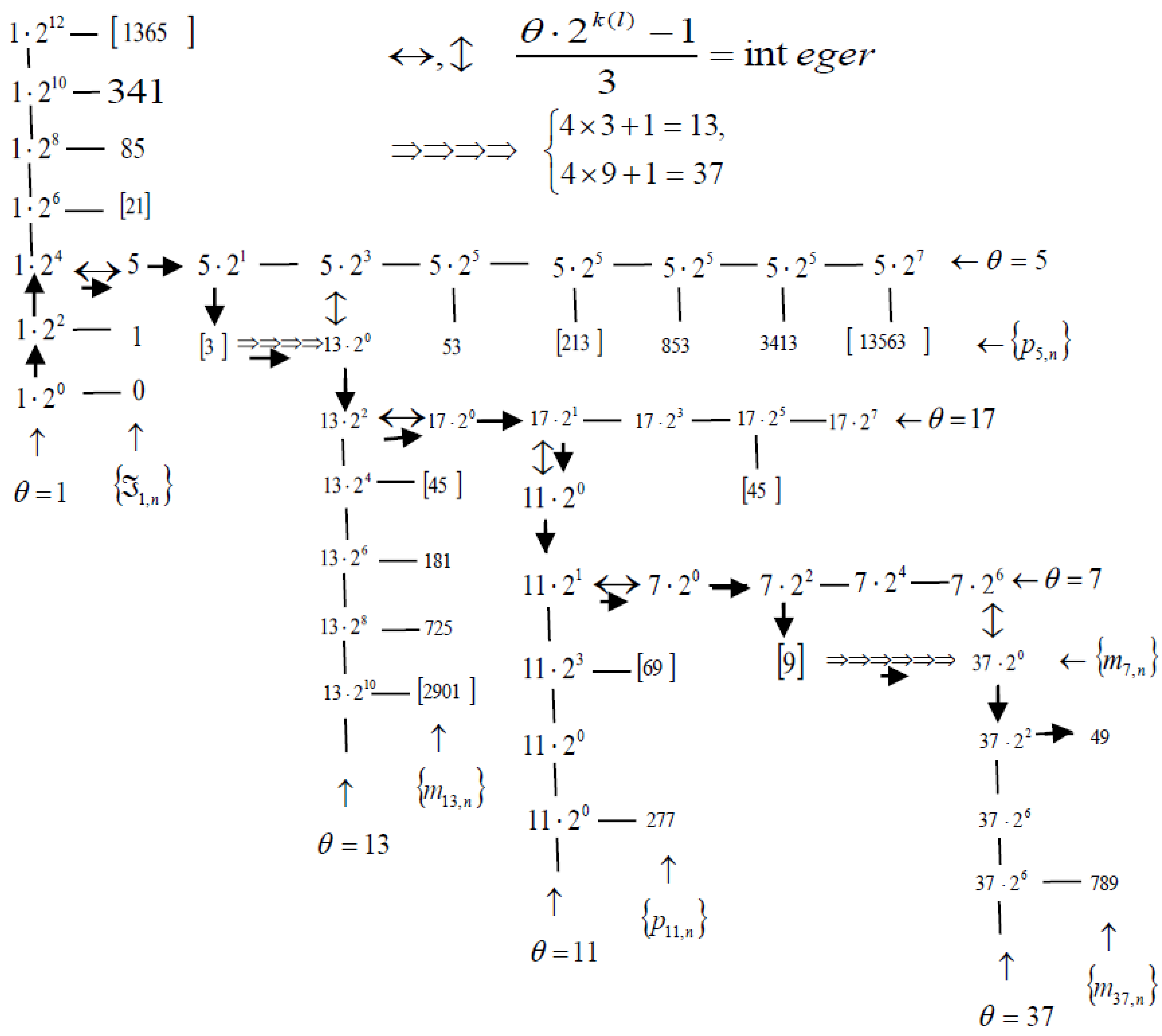

Figure 2:

References

[1] A. Ahmed. *Two different scenarios when the Collatz Conjecture fails*, Gen. Lett. Math., **11**, 2 (2021), 46–54 https://doi.org/10.31559/glm2021.11.2.4 1, 1

[2] *L.Collatz On the motivation and origin of the (3n + 1) Problem*, J. Qufu Normal University, Natural Science Edition, **12**, 3 (1986), 9–11 1