Available online at www.refaad.com
Uncertainty, 1 (1) (2024), 11-16
Research Article

Uncertainty

Journal Homepage: https://www.refaad.com/Journal/Index/21

Q- Rung Orthopair Fuzzy Sets and Topological Spaces

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Abstract

The concept of q-rung orthopair fuzzy set, where q is a positive integer, introduced by Yager, is studied in the present paper and fundamental properties of it are examined. The concept of the 1-rung orthopair fuzzy set coincides with Atanassov's intuitionistic fuzzy set, a 2-rung orthopair fuzzy set is known as a Pythagorean fuzzy set, while a 3-rung orthopair fuzzy set is referred to as a Fermatean fuzzy set. Also the ordinary notion of topological space is extended in this work to a q-rung orthopair fuzzy environment, as well as the fundamental properties and concepts of convergence, continuity, compactness and of Hausdorff topological space. All these contents are illustrated by suitable examples.

Keywords: uncertainty; fuzzy set; intuitionistic fuzzy set; Pythagorean fuzzy set; Fermatean fuzzy set; q-rung orthopair fuzzy set; q-rung orthopair fuzzy topological space.

2020 MSC: 03E72.

1 Introduction

Zadeh, in 1965, extended the concept of a crisp set to that of a *fuzzy set* [1], on the purpose of tackling mathematically the existing in everyday life partial truths (e.g. rather good, almost equal, etc.) , as well as the definitions having no clear boundaries, like "high mountains", "clever people", etc. Zadeh's idea was to replace the objective function of a crisp set with the membership function in a fuzzy set, which takes values in the interval [0, 1]. In this way a membership degree between 0 and 1 is assigned to each element of the universal set with respect to the corresponding fuzzy set.

Before the introduction of fuzzy sets, probability used to be the unique mathematical tool in hands of the experts for managing the existing in real world uncertainty, caused by the shortage of knowledge about an observed phenomenon. Several types of uncertainty appear in everyday life, including *randomness, imprecision, vagueness, ambiguity, inconsistency*, etc. [2]. The uncertainty due to randomness is related to well-defined events whose outcomes cannot be predicted in advance, like the turn of a coin, the throwing of a die, etc. Imprecision occurs when the corresponding events are well defined, but the possible outcomes cannot be expressed with an exact numerical value; e.g. "The temperature tomorrow will be over 20^{o} C". Vagueness is created when one is unable to clearly differentiate between two properties, like a good and a mediocre student. In case of ambiguity the existing information leads to several interpretations by different observers. For example, the phrase "Boy no girl" spelled as "Boy, no girl" means boy, but spelled as "Boy no, girl" means girl. Inconsistency appears when two or more pieces of information cannot be true at the same time. As a result the obtainable in this case

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doi: https://doi.org/10.31559/uncertainty2024.1.1.2
Received: 18 Mar 2024 Revised: 1 May 2024 Accepted: 1 June 2024

information is conflicted or undetermined. For example, "The chance of raining tomorrow is 80%, but this does not mean that the chance of not raining is 20%, because they might be hidden weather conditions".

Probability, however, was proved to be effective only for tackling the uncertainty due to randomness, in contrast to fuzzy sets which were proved to be suitable for tackling other forms of uncertainty as well, and in particular the uncertainty due to vagueness [3]. Following the introduction of fuzzy sets, several generalizations of them and theories related to them have been proposed on the purpose of tackling more effectively all the forms of the existing uncertainty, e.g. see [4]. None of these generalizations or theories, however, was proved to be suitable for tackling alone all the forms of the uncertainty alone, but the synthesis of all of them forms an effective framework for this purpose.

In 1986, Atanassov expanded fuzzy set to the concept of *intuitionistic fuzzy set* by adding the degree of non-membership to Zadeh's degree of membership [5]. Intuitionistic fuzzy sets can be used everywhere the ordinary fuzzy sets can be applied, but this is not always necessary. An example where the use of intuitionistic fuzzy sets is necessary is the case of an election, where a candidate can have voted for (membership), or voted against (non-membership) by the electoral vote. The intuitionistic fuzzy sets are suitable for tackling the uncertainty due to imprecision, which appears frequently in human reasoning [6].

Yager introduced later the concept of the *q-rung orthopair fuzzy set,* where q is a positive integer, in which the sum of the q-th powers of the membership and non-membership degrees of the elements of the universal set is bounded by 1 [7]. The concept of a 1-rung orthopair fuzzy set coincides with that of intuitionistic fuzzy set, a 2-rung orthopair fuzzy set is referred to as a *Pythagorean fuzzy set* [8] and 3-rung orthopair fuzzy set is referred to as a *Fermatean fuzzy set* [9]. It has been established that Pythagorean and Fermatean fuzzy sets have stronger ability than intuitionistic fuzzy sets for tackling the uncertainty in decision-making problems [10].

Notice also that in 1995 Smarandache, drawing inspiration from the neutralities that frequently surface in everyday life, like <friend, neutral, enemy>, <win, draw, defeat>, <high, medium, short>, etc., added the degree of indeterminacy or neutrality and expanded intuitionistic fuzzy set to the concept of *neutrosophic set* [11]. Neutrosophic sets are effective for tackling the uncertainty due to inconsistency and ambiguity. Recently the concept of Pythagorean fuzzy set was extended to neutrosophic environments [12] and the same happened with the Fermatean fuzzy set [13].

In this work we study the concept of the q-rung orthopair fuzzy set focusing on fundamental properties of it and on extending the ordinary notion of topological space to the notion of a *q-rung orthopair fuzzy topological space.* The rest of the paper is formulated as follows: Section 2 contains the necessary mathematical background for the understanding of the paper. The concept of the q-rung orthopair fuzzy set is presented in Section 3 together with basic properties of these sets. In Section 4 the classical notion of topological space is extended to *q-rung orthopair fuzzy topological spaces* together with fundamental properties and concepts like convergence, continuity, compactness and Hausdorff topological spaces. The paper closes with the final conclusions and some hints for further research included in its last Section 5.

2 Mathematical Background

The exact definition of a fuzzy set [1] is the following:

Definition 1: A fuzzy set A in the universal set of the discourse U is a set of ordered pairs of the form:

$$A = \{(x, m(x)): x \in U, m(x) \in [0, 1]\} \quad (1)$$

In equation (1), m: U → [0, 1] is the membership function of A, and the real value y = m(x) is the membership degree of x in A, for all x in U. The greater m(x), the better x satisfies the characteristic property of A.

With the help of Definition 1, the basic concepts and operations defined on crisp sets can be extended in a natural way to fuzzy sets [2]. For example, the membership function of the *empty* fuzzy set $\emptyset_U$ in U is defined by m(x) = 0 and of the universal fuzzy set I_U by m(x) = 1, ∀ x ∈ U. Further, if A and B are fuzzy sets in U, with membership functions m_A, m_B respectively, then:

- A is called a *subset* of B (A ⊆ B), if, and only if, $m_A(x) \le m_B(x)$, ∀ x ∈ U. If we have simultaneously A ⊆ B and B ⊆ A, then A and B are called *equal* fuzzy sets (A=B).
- The *complement* of A is the fuzzy set A^C in U with membership function equal to 1 – m_A.
- The *union* A∪B is the fuzzy set C in U with membership function defined by $m_C(x)$ =max {$m_A(x)$, $m_B(x)$], ∀ x ∈ U.
- The *intersection* A∩B is the fuzzy set D in U with membership function defined by $m_C(x)$ =min {$m_A(x)$, $m_B(x)$], ∀ x ∈ U.

If A and B are crisp sets, it is straightforward to check that the previous definitions are reduced to the corresponding ordinary definitions for crisp sets. It is also straightforward to check that most of the laws and properties of crisp sets are also true for fuzzy sets, like the commutative and associative laws for the union and intersection, the distributive law of the union with respect to intersection and vice versa, the double complement property $(A^C)^C = A$, etc.

The definition of the membership function of a fuzzy set A is not unique depending on the personal goals of each observer. For example, if A is the fuzzy set of "tall men", one may consider all the men with heights greater than 1.90 m as being tall and another one all those with heights greater than 2 m. As a result, the first observer will assign membership degree 1 to all men with heights between 1.90 m and 2 m, whereas the second one will assign membership degrees <1 to them. The only restriction for the definition of the membership function is to be compatible with common sense; otherwise, it does not give a reliable description of the real situation represented by the corresponding fuzzy set. This could happen for instance in the previous example, if men with heights less than 1.60 m had membership degrees > 0.5. The difficulty, however, with the definition of the membership function did not prevent the theory of fuzzy sets to find many and important applications to almost all sectors of human activity, e.g. see [2], [3], etc.

Several efforts have been made to overcome the previous difficulty with the membership function: The *interval-valued fuzzy sets* [14], for example, replace the membership degrees of a fuzzy set with subintervals of [0, 1] and the *hesitant fuzzy sets* [15] replace them with subsets of [0, 1]. Also, in the *rough sets* [16] the membership function of the fuzzy set is replaced by a pair of sets which give the lower and the upper approximation of the original set, in the *soft sets* [17] the existing uncertainty is tackled in a parametric manner, etc.
Atanassov extended fuzzy set to the concept of intuitionistic fuzzy set [5] as follows:
Definition 2: An intuitionistic fuzzy set A in the universal set U is of the form

$$A = \{(x, m(x), n(x)): x \in U, m(x), n(x) \in [0, 1], \quad 0 \leq m(x) + n(x) \leq 1\} \tag{2}$$

In equation (2) m: U → [0, 1] is the membership function, n: U → [0, 1] is the non-membership function of A and m(x), n(x) are the degrees of membership and non-membership respectively, for each x in A. For simplicity we write A = <m, n>. Further, h(x) = 1 - m(x) - n(x), is said to be the degree of *hesitation* of x in A. If h(x) = 0, then A is a fuzzy set.

The term intuitionistic fuzzy set is due to Atanassov's collaborator Gargov [18] in analogy to the idea of intuitionism introduced by Brouwer at the beginning of the last century [19]. It is recalled that intuitionism rejects Aristotle's law of the excluded middle by stating that a proposition is either true, or not true, or we do not know if it is true or not true. The first part of this statement corresponds to Zadeh's membership degree, the second to Atanassov's non-membership and the third to the degree of hesitation.
Example 1: Let U be the set of the students of a class, let A be the intuitionistic fuzzy set of the good students of the class. Then each student x of U is characterized by an *intuitionistic fuzzy pair* (m, n) with respect to A, with m, n in [0, 1].For example, if (x, 0.5, 0.3) is in A, then, there is a 50% belief that x is a good student, but also a 30% belief that he is not a good student and a 20% hesitation to characterize him as a good student or not.

The properties and operations of fuzzy sets can be extended to intuitionistic fuzzy sets [6]. For example, if $A = <m_A, n_A>$ and $B = <m_B, n_B>$ are intuitionistic fuzzy sets in U, their union A∪B is the jntuitionistic fuzzy set $C = <m_C, n_C>$ in U with $m_C(x)= \max \{m_A(x), m_B(x)\}$ and $n_C)x) = \min \{n_A(x), n_B(x)\}$, ∀ x ∈ U. The difficulty with the definition of the membership and non-membership functions, however, remains the same as in fuzzy sets.

3 The Concept and Basic Properties of the q- Orthopair Fuzzy Set

When defining the membership and non-membership degrees of the elements of the universal set U, it could happen that m(x) + n(x) >1. In such cases the corresponding structure cannot be treated as an intuitionistic fuzzy set. This motivated Yager to define the wider class of q-rung orthopair fuzzy sets [7] as follows:
Definition 3: A q-rung orthopair fuzzy set A in the universal set U, where q is a positive integer, is of the form

$$A = \{(x, m(x), n(x)): x \in U, m(x), n(x) \in [0, 1], \quad 0 \leq [m(x)]^q + [n(x)]^q \leq 1\} \tag{3}$$

In equation (3) m(x) is the membership and n(x) is the non-membership degree of x in A respectively.

The concept of 1-rung orthopair fuzzy set coincides with the concept of intuitionistic fuzzy set. Further, a 2-rung orthopair fuzzy set is referred to as a Pythagorean fuzzy set [8] and a 3-rung orthopair fuzzy set is referred to as a Fermatean fuzzy set [9].

The following Proposition helps to clarify the Yager's motivation for introducing the notion of the q-rung orthopair fuzzy set:
Proposition 1: Let q_1, q_2 be positive integers, with $q_2 > q_1$. Then the set $\{(m, n): m, n \in [0, 1], 0 \leq m^{q_2} + n^{q_2} \leq 1\}$ is larger than the set $\{(m, n): m, n \in [0, 1], 0 \leq m^{q_1} + n^{q_1} \leq 1\}$.
***Proof*:** Since m, n ∈ [0, 1], is $m^{q_2} + n^{q_2} < m^{q_1} + n^{q_1}$. Consequently, if $m^{q_1} + n^{q_1} \leq 1$, it is also $m^{q_2} + n^{q_2} \leq 1$ and the result follows.

Example 2: Let (x, 0.8, 9.7) be an element of the q-rung orthopair fuzzy set A. Then, since 0.8 + 0.7 >1, A is not an intuitionistic fuzzy set, Also, since $(0.8)^2 + (0.7)^2 = 0.64 + 0.49 > 1$, A is not a Pythagorean fuzzy set too. But $(0.8)^3 + (0.7)^3 = 0.512 + 0.343 < 1$. Thus A could be a Fermatean fuzzy set, this depending on the form of its other elements.

Proposition 2: Let A be q_1-rung and B be q_2-rung orthopair fuzzy sets respectively, with $q_2 > q_1$. Then A is also a q_2-rung orthopair fuzzy set.

***Proof*:** Let x(m, n) be an element of A. Then $0 \leq m^{q_1} + n^{q_1} \leq 1$, with m, n ∈ [0, 1]. But $q_2 > q_1$, therefore, $0 \leq m^{q_2} + n^{q_2} \leq m^{q_1} + n^{q_1} \leq 1$ and the result follows.

In particular, an intuitionistic fuzzy set is a Pythagorean fuzzy set and a Pythagorean fuzzy set is a Fermatean fuzzy set

The classical operations on crisp sets can be generalized for q-rung orthopair fuzzy sets. Here we define the subset and the complement of a q-rung orthopair fuzzy set, as well as the union and intersection of two such sets.

Definition 4: Let A = $\langle m_A, n_A \rangle$ and B = $\langle m_B, n_B \rangle$ be two q-rung orthopair fuzzy sets in the universe U. Then:

i) A is called a *subset* of B ($A \subseteq B$), if, and only if, $m_A(x) \leq m_B(x)$ and $n_A(x) \geq n_B(x)$, ∀ x ∈ U. If we have simultaneously $A \subseteq B$ and $B \subseteq A$, then A and B are called *equal* q-rung orthopair fuzzy sets (A=B).

ii) The *complement* of A = $\langle m_A, n_A \rangle$ is the q-rung orthopair fuzzy set $A^C = \langle n_A, m_A \rangle$ in U.

iii) The *union* A∪B is the q-rung orthopair fuzzy set C = $\langle m_C, n_C \rangle$ in U with $m_C = \max\{m_A, m_B\}$ and $n_C = \min\{n_A, n_B\}$.

iv) The *intersection* A∩B is the q-rung orthopair fuzzy set D = $\langle m_D, n_D \rangle$ in U with $m_D = \min\{m_A, m_B\}$ and $n_C = \max\{n_A, n_B\}$.

Remark 1: i) It is easy to check that all the above relations are well defined. For the union, for example, set m = $\max\{m_A, m_B\}$ and n = $\min\{n_A, n_B\}$. If $m = m_A$ and $n = n_B$, then $0 \leq m^q + n^q = (m_A)^q + (n_B)^q \leq (m_A)^q + (n_A)^q \leq 1$. In an analogous way one can show that we always have $m^q + n^q \leq 1$ for all the other possible combinations, which means that A∪B is a q-rung orthopair fuzzy set.

ii) If A and B are crisp sets, it is straightforward to check that the previous definitions are reduced to the corresponding ordinary definitions for crisp sets.

iii) With the help of the previous definitions it is straightforward to check that most of the laws and properties of crisp sets are also true for q-rung orthopair fuzzy sets, like the commutative and associative laws for the union and intersection, the distributive law of the union with respect to intersection and vice versa, the double complement property $(A^C)^C = A$, etc.

Example 3: Let U= $\{x_1, x_2, x_3\}$ be the universal set and let A=$\{(0.3, 0.6, x_1),(0.5, 0.4, x_2), (0.7, 0.5, x_3)\}$and B=$\{(0.6, 0.2, x_1), (0.3, 0.5, x_2), (0.3, 0.6,x_3)\}$be two q-rung orthopair fuzzy sets in U, q>1. Then:

i) Neither $A \subseteq B$, nor $B \subseteq A$

ii) $A^C = \{(0.6, 0.3, x_1), (0.4, 0.5, x_2), (0.5, 0.7, x_3)\}$ and $B^C = \{(0.2, 0.6, x_1), (0.5, 0.3, x_2) (0.6, 0.3, x_3)\}$

iii)A∪B = $\{(0.6, 0.2, x_1), (0.5, 0.4, x_2), (0.7, 0.5, x_3)\}$

iv)A∩B = $\{(0.3, 0.6, x_1), (0.3, 0.5, x_2), (0.3, 0.6, x_3)\}$

Definition 5: i) The *empty* q-rung orthopair fuzzy set in the universe U is defined to be $\emptyset_U = \{(x, 0, 1): x \in U\}$.

ii) The *universal* q-rung orthopair fuzzy set in U is defined to be $I_U = \{(x, 1, 0): x \in U\}$.

It is straightforward to check that for each q-rung orthopair fuzzy set A in U is $A \cup I_U = I_U$, $A \cap I_U = A$, $A \cup \emptyset_U = A$ and $A \cap \emptyset_U = \emptyset_U$.

4 q-Rung Orthopair Fuzzy Topological Spaces

Topological spaces are the most general category of mathematical spaces, on which fundamental properties like convergence, continuity, compactness, etc. are defined [20]. The ordinary notion of topological space has been extended to fuzzy [21], to intuitionistic fuzzy [22], to soft [23], to neutrosophic topological space [24], etc. For example, a fuzzy topological space is defined as follows:

- A *fuzzy topology* T on a non-empty set U is defined as a collection of fuzzy sets in U such that:

1. I_U and $\emptyset_U$ belong to T
2. The intersection of any two elements of T belongs to T
3. The union of any number (finite or infinite) of elements of T belongs also to T

The elements of a fuzzy topology Ton U are called *open* fuzzy sets of U and their complements are called *closed* fuzzy sets of U. The pair (U, T) is referred to as a *fuzzy topological space* on U.

Here we extend the notion of topological space to the notion to *q-rung orthopair fuzzy topological space* and we study the previously mentioned properties on such kind of spaces.

Definition 6: A *q-rung orthopair fuzzy topology* T on a non-empty set U is defined as a collection of q-rung orthopair fuzzy sets in U such that:

4. I_U and $\emptyset_U$ belong to T
5. The intersection of any two elements of T belongs to T
6. The union of any number (finite or infinite) of elements of T belongs also to T.

Trivial examples are the *discrete q-rung orthopair fuzzy topology* of all q-rung orthopair fuzzy sets in U and the *non-discrete orthopair q-rung fuzzy topology* T= $\{I_U, \emptyset_U\}$.
The elements of a q-rung orthopair fuzzy topology Ton U are called *open* q-rung orthopair fuzzy sets of U and their complements are called *closed* q-rung orthopair fuzzy sets of U. The pair (U, T) is referred to as a *q-rung orthopair fuzzy topological space* on U.
Example 4: Let U = {x} and let A = {(x , 0.5, 0.4)}, B = {(x , 0.4, 0.8)}, C = {(x , 0.5, 0.4)}, D ={(x , 0.4, 0.8)} be q-rung orthopair fuzzy sets in U, q > 1. Then it is straightforward to check that the collection T = $\{\emptyset_U, I_U, A, B, C, D\}$ is a q-rung orthopair fuzzy topology on U.
We close by extending the concepts of convergence, continuity, compactness and of Hausdorff topological space to q-rung orthopair fuzzy topological spaces.
Definition 7: Given two q-rung orthopair fuzzy sets A and B of the q-rung orthopair fuzzy topological space (U, T), B is said to be a *neighborhood* of A, if there exists an open q-rung orthopair fuzzy set Q such that $A \subseteq Q \subset B$. Further, we say that a sequence $\{A_n\}$ of q-rung orthopair fuzzy sets of (U, T) *converges* to the q-rung orthopair fuzzy set A of (U, T), if there exists a positive integer m such that for each integer n≥m and each neighborhood B of A we have that $A_n \subset B$.
The following Proposition generalizes Zadeh's *extension principle* for fuzzy sets (see [2], pp. 20-21) to q-rung orthopair fuzzy sets
Proposition 3: Let U and V be two non-empty crisp sets and let g: U→V be a function. Then g can be extended to a function G mapping q-rung orthopair fuzzy sets of U to q-rung orthopair fuzzy sets of V.
Proof: Let A$<m_A, n_A>$ be a q-rung orthopair fuzzy set of U. Then its image G(A) is a q-rung orthopair fuzzy set B of V, whose components are defined as follows: Given y in V, consider the set $g^{-1}(y)=\{x \in U: g(x)=y\}$. If $g^{-1}(y) = \emptyset$, then $m_B(y)=0$, and if $g^{-1}(v)\neq\emptyset$, then $m_B(y)$ is equal to the maximal value of all $m_A(x)$ such that $x \in g^{-1}(y)$. Conversely, the inverse image $G^{-1}(B)$ is the q-rung orthopair fuzzy set A of U with membership function $m_A(x)=m_B(g(x)$, for each $x \in U$. In an analogous way one can determine the component n_B of B.
Definition 8: Let (U, T) and (V, S) be two q-rung fuzzy topological spaces on the non-empty crisp sets U and V respectively and let g be a function g: U→V. Then, according to Proposition 3, g can be extended to a function G which maps q-rung orthopair fuzzy sets of U to q-rung orthopair fuzzy sets of V. We say then that g is a *q-rung orthopair fuzzy continuous* function, if, and only if, the inverse image of each open q-rung orhopair fuzzy set of V through G is an open q-rung orthopair fuzzy set of U.
Definition 9: A family A = $\{A_i, I \in I\}$ of q-rung orthopair fuzzy sets of the q-rung orthopair fuzzy topological space (U, T) is called a *cover* of U, if U = $\bigcup_{i\in I} A_i$. If the elements of A are open q-rung orthopair fuzzy sets, then A is called an *open cover* of U. Also, each subset of A which is also a cover of U is called a sub-cover of A. The q-rung orthopair fuzzy topological space (U, T) is said to be *compact*, if every open cover of U contains a sub-cover with finitely many elements.
Definition 9: A q-rung orthopair fuzzy topological space (U, T) is called a *T_1- q-rung orthopair fuzzy topological space* if, and only if, for each pair of elements x_1, x_2 of U with $x_1 \neq x_2$, there exist at least two open q-rung orthopair fuzzy sets Q_1 and Q_2 such that $x_1 \in Q_1$, $x_2 \notin O_1$ and $x_2 \in Q_2$, $x_1 \notin O_2$.
Definition 10: A q-rung orthopair fuzzy topological space (U, T) is called a *T_2- q-rung orthopair fuzzy topological space* if, and only if, for each pair of elements x_1, x_2 of U with $x_1 \neq x_2$, there exist at least two open q-rung orthopair fuzzy sets Q_1 and Q_2 of U such that $x_1 \in O_1$, $x_2 \in O_2$ and $O_1 \cap O_2 = \emptyset_U$.

A T_2-q-rung orthopair fuzzy topological space is also called a *Hausdorff* or a *separable* q-rung orhopair fuzzy topological space. Obviously a T_2-rung orthopair fuzzy topological space is always a T_1- q-rung orthopair fuzzy topological space.

5 Conclusion

In this work we studied the concept of q-rung orthopair fuzzy set and we extended the classic notion of topological space and the fundamental properties of convergence, continuity, compactness and Hausdorff space to q-rung orthopair fuzzy topological spaces. Examples were also given to illustrate our results.

It looks that proper combinations of the theories developed for tackling the existing in real life uncertainty is a promising tool for obtaining better results in a variety of human activities characterized by uncertainty, like assessment and decision-making [25]-[27]. This is, therefore, a fruitful area for future research.

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