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Countably and Locally Compactness in Bitopological Spaces

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Abstract

The main objectives of this paper are to introduce the concept of pairwise countably compact in bitopological space,and we introduce some generalization of pairwise countably compact in bitopological spaces, and some of their properties, and relate it to other spaces.

Keywords: pairwise compact, pairwise countably compact, pairwise locally compact, bitopological space, pairwise open cover.
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1. Introduction

In this paper we apply bitopological concepts to introduce definitions for generalized countable compact space, generalized compact space and generalized almost compact space and nearly compact space in bitopological spacesr. The basic concepts of some generalized compact space and generalized almost compact space and nearly compact space are sufficiently illustrated. Moreover, proved results, examples and counter examples are provided.
From 1963, when Kellay introduced the concept of bitopological space, several topological properties, which are already included in a single topology, are generalized into bitopological spaces. Some of these properties are compactness, paracompactness, separation axioms, connectedness, some special types of functions and many others. Many authors studied and investigated these bitopological spaces after Kelly, like Fletcher . (1969), Birsan (1969) Reilly (1970) , Datta (1972) , Hdeib and Fora (1982, 1983) , Bose et al. (2008), Killiman and salleh (2011) , Abushaheen et al. (2016) , and Qoqazeh et al. (2018).One of the most powerful notions in system analysis is the concept of topological structures and their generalizations. Rough set theory, introduced by Pawlak in 1982, is a mathematical tool that supports also the uncertainty reasoning but qualitatively. In this thesis, we shall integrate some ideas in terms of concepts in topology. Topology is a branch of mathematics, whose concepts exist not only in almost all branches of mathematics, but also in many real-life applications. We believe that topological structure will be an important base for modification of knowledge extraction and processing.

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2. Basic concepts of the compactness in bitopological spaces

In this section, we will introduce the basic concepts of the compactness in bitopological spaces and give several results.

Definition 2.1. *[4] A bitopological space (χ, β_1, β_2) is called pairwise compact if every $(P-$open) cover of space has a finite subcover .*

Definition 2.2. *[4] Let (χ, β_1, β_2) be a topologcal space, for each A subset of X is called that pairwise closed ($P - \text{closed})$ in X if there is β_1-closed set and β_2-closed set.*

$A \subset X$ is called pairwise open $(P-$open) if $X - A$ is pairwise closed in X.

Definition 2.3. *[7] A cover $\tilde{U}$ of the bitopological space (χ, β_1, β_2) is called $\beta_1\beta_2-$ is pairwise open cover if $\tilde{U} \subset \beta_1 \cup \beta_2$.*

Definition 2.4. *[8] Let $\tilde{A}$ be an open cover of a bitopological space (χ, β_1, β_2), $\tilde{A}$ we say that pairwise open cover if $\tilde{A} \subset \beta_1 \cup \beta_2$ and $A \cap \beta_1$ a non empty set and $A \cap \beta_2$ a non empty set.*

Remark 2.1. *[2] Every finite space in pairwise topological space is compact.*

Theorem 2.2. *[2] Every finite subset of any pairwise topological space is compact.*

Definition 2.5. *[7] Let $X = (\chi, \beta_1, \beta_2)$ be a bitopological space, X we say that semi—compact (S—compact) if every β_1-open cover and β_2-open cover of the space (χ, β_1, β_2) has a finite subcover.*

Definition 2.6. *[2] Let β_1 and β_2 be two topologies on X. Then $\beta_1 \cup \beta_2$ forms a subbase for some topology on X; this topology is called the least upper bound (L.U.B) topology and denoted by $(\chi, \langle\beta_1, \beta_2\rangle)$. Each basic open set B in $(\chi, \langle\beta_1, \beta_2\rangle)$ has the form $B = \bigcap_{i=1}^{n} B_i$ where $B_i \in \beta_1$ or $B_i \in \beta_2$ for all $i = 1, 2, ..., n$. The intersection of the B_i 's which are in β_1, and The intersection of the B_i 's which are in β_2,. So $B = U \cap V$, where $U \in \beta_1$and $V \in \beta_2$.*

Definition 2.7. *[2] Let A be a subset of $(\chi, \langle\beta_1, \beta_2\rangle)$, then A is called to be (S—open) in the least upper bound (L.U.B) topology $(\chi, \langle\beta_1, \beta_2\rangle)$.*

Theorem 2.3. *[2] A bitopological space (χ, β_1, β_2) is called semi-compact (S-compact) if and only if $(\chi, \langle\beta_1, \beta_2\rangle)$ is compact.*

Proof. Let (χ, β_1, β_2) then it is semi-compact (S-compact), and let $\tilde{U}$ be any $\langle\beta_1, \beta_2\rangle$ open cover for X consisting of sub-base open sets. Then $\tilde{U} \subset \beta_1 \cup \beta_2$, so $\tilde{U}$ is a $\beta_1\beta_2-$open cover of X. Thus $\tilde{U}$ has a finite subcover for X. Hence $(\chi, \langle\beta_1, \beta_2\rangle)$ is compact. Conversely, let $(\chi, \langle\beta_1, \beta_2\rangle)$ be a compact space and $\tilde{V}$ be a $\beta_1\beta_2-$open cover of X. Then $\tilde{V} \subset \beta \cup \beta_2 \subset \langle\beta_1, \beta_2\rangle$. So $\tilde{V}$ has a finite subcover for X. □

Definition 2.8. *[2] For each A subset of $(\chi, \langle\beta_1, \beta_2\rangle)$ is S–compact if every cover of A by (S—open) subsets of X has a finite subcover.*

The following theorems are easy concerning S–compact.

Theorem 2.4. *[6] Every S—closed subset of a S—compact bitopological space (χ, β_1, β_2) is S—compact.*

Theorem 2.5. *[6] Every S—compact subset of a $(P - T_2)$ bitopological space (χ, β_1, β_2) is S—closed. (i.e. closed in $(\chi, \langle\beta_1, \beta_2\rangle)$).*

Theorem 2.6. *[5] The union of two S—compact subspaces of a $(\chi, \langle\tau_1, \tau_2\rangle)$ space is S—compact.*

Definition 2.9. *[3] In a topological space (χ, β_1, β_2) a cover $\tilde{U}$ is called $\beta_1\beta_2-$open if $\tilde{U} \subset \beta_1 \cup \beta_2$, if in addition, $\tilde{U}$ contains at least one non-empty member of β_1 and at least one non-empty member of β_2, it we say that P—open.*

A bitopological space (χ, β_1, β_2) is called pairwise compact (P—compact) if every pairwise open cover of the space (χ, β_1, β_2) has a finite subcover.

The following theorems give the relation between the previous types of compactness in bitopological spaces.

Theorem 2.7. *[7] If (χ, β_1, β_2) is a P-compact, then it is S-compact .*
the convers of theorem need not be true , for example :

Example 2.1. *[7] $(\mathbb{R}, \beta_l, \beta_r)$. Then X is P—compact, but not S—compact.*

Example 2.2. *[8] Let $\chi = [0,1]$, $\beta_1 = \{\chi, \phi\} \cup \{[0,b) : b \in X\}$ and $\beta_2 = \{\chi, \phi, \{1\}\}$. Then (χ, β_1, β_2) iscompactbutnotS—c*

Lemma 2.8. *[6] If (χ, β_1, β_2) is a topological space and A is a subset of X, then the collection $\beta(A)$ gives by $\{\phi, X\} \cup \{A \cup U / U \in \beta\}$ is a topology on X. this topology called the ad-joint topology of A.*

Theorem 2.9. *[6] For a bitopological space (χ, β_1, β_2), the following are equivalent :*
(i) (χ, β_1, β_2) *is P—compact.*
(ii) *Let m be a β_i-open set then the toplogey β_j is compact ; for all $i \neq j$; $i, j = 1, 2$.*
(iii) *Every β_i-closed proper subset of y is a β_j-compact, for all $i \neq j$, $i, j = 1, 2$.*

(Ai-Refa′ei, 1985) introduced some theorems on a P—compact subset of a bitopological space.

Theorem 2.10. *[1] Let $y = (\chi, \beta_1, \beta_2)$ a $P - T_2$—space, and m be a P–compact subset of y. Then m is closed subset in $(\chi, \langle\beta_1, \beta_2\rangle)$, i.e. m is S–closed.*

(Birsan, 1969) birsan important definitions of compactness in bitoplojical spaces.

Definition 2.10. *[1] Let $y = (\chi, \beta_1, \beta_2)$ be a topological space then y called β_j-compact with respect to β_j if for each β_i-open cover of y,there is a finite $\beta - j$—open, for $i \neq j$, $i, j = \{1, 2\}$.*

A bitopological space $(\chi, \beta 1, \beta 2)$ we say that B—compact if it is $\beta 1$—compact with respect to β_2 and β_2—compact with respect to β_1.

The following examples show that there is no relation between B—compact, S—compact and P—compact.

Example 2.3. *[1] (1) Let $y = \{a, b, c\}$ with $\beta_1 = \{\phi, , \{a\}, \{b, c\}\}$ and $\beta_2 = \{\phi, , \{a\}, \{b, c\}\}$. Then $y = (\chi, \beta_1, \beta_2)$ is S–compact and P–compact, since y is finite. But (χ, β_1, β_2) is not B—compact, since the β_1—open cover $\{\phi, \chi, \{a\}, \{b, c\}\}$ has no β_2—open subcover.*
(2) Let $y = [a, b]$ with $\beta_1 = \{\phi, y, \{a\}\} \cup \{[a, m] : m \in y\}$ and $\beta_2 = \{\phi, y, \{b\}\} \cup \{(a, 1] : a \in y\}$. Then $y = (\chi, \beta_1, \beta_2)$ is B—compact. However (χ, β_1, β_2) is neither S—compact nor P—compact. since y is finite. Consider the P—open cover $\{a\} \cup \{(m, a] : m \in X, m \neq a\}$ has no finite subcover.

Birsan characterized the concept of β_1—compact with respect to β_2 as we will see in the following theorems.

Theorem 2.11. *[1] For a bitopological space (χ, β_1, β_2), the following are equivalent :*
(i) (χ, β_1, β_2) *is β_1–compact with respect to β_2.*
(ii) *For any family $\tilde{M} = \{N_\alpha : \alpha \in \Delta\}$ of β_1—closed sets which has the empty intersection, there exists a finite family $\{G_j : j = 1, 2, ..., n\}$ of β_2—closed sets with empty intersection and satisfies the condition that for all $j = 1, 2, ..., n$, there exists $\alpha_i \in \Delta$, such that $F\alpha_j \subset G_J$ has nonempty intersection, such that $\tilde{M}$ has nonempty intersection.*
(iii) *For any family $\tilde{M} = \{N_\alpha : \alpha \in \Delta\}$ ofβ_1—closed sets with the property that every finite family $\{G_j : j = 1, 2, ..., n\}$ of β_2—closed sets with empty intersection and satisfies the condition that for all $j = 1, 2, ..., n$, there exists $\alpha_i \in \Delta$, such that $N\alpha_j \subset G_J$ has nonempty intersection such that $\tilde{M}$ has nonempty intersection.*

3. Countably Compactness in bitopological spaces

Definition 3.1. *A bitopological space* (χ, β_1, β_2) *we say that* P–*countably compact if every countable* P—*open cover of the space* (χ, β_1, β_2) *has a finite subcover.*

Definition 3.2. *A bitopological space* (χ, β_1, β_2) *we say that* S—*countably compact if ever countable* $\beta_1\beta_2$—*open cover of the space* (χ, β_1, β_2) *has a finite subcover.*

Definition 3.3. *B collection* $\mathcal{B}$ *of nonvoid* β_1 *or* β_2 *closed set in* Y *is pairwise closed if* $B_1 \subset \mathcal{B}$ *and* $B_2 \subset \mathcal{B}$ *such that* B_1 *is a* B_1*- closed proper subset of* Y *and* B_2 *is* β_2 *- closed proper subset of* Y.

Definition 3.4. *Let* $\mathcal{B}$ *be a collection of* Y *then we say* $\mathcal{B}$ *is filter base on* Y *if* $B_1 \subset \mathcal{B}$, $B_2 \subset \mathcal{B}$ *such that* B *and* B*is proper subset of* Y.

Definition 3.5. *A net* $\{S_n : n \in D\}$ *in* y *is called pairwise net if there exist* m *and* n *in* D *such that* $\{\overline{S_k :\geqslant n}\}$ *and* $\{\overline{S_k :\geqslant m}\}$ *are proper subset of* y.

Theorem 3.1. *The following statements on a bitopological space are equivalent :*
(a) (χ, β_1, β_2) *is is pairwise compact.*
(b) *Every* P—*closed family of subsets* y *with finite intersection property has non void intersection.*
(c)) *Every pairwise filter base (Pairwise net) has at least one* β_1 *and* β_2*-accumulation point.*
(d) *Every pairwise maximal filter base has at least one* β_1 *and* β_2 *- limit point.*

proof 1. *(a) implies (b) : This part is obvious from the fact that the complement of pairwise* Closed *family is pairwise open.*
(b) implies (d) : Let $\mathcal{A}$ *be pairwise maximal filter base on* Y *then the family* $\{N : F \in\} \cup \{F^{\beta:F\in\beta_2\}}$ *is a* P—*closed family with finite intersection property. By the hypothesis, we have* $(\bigcap\{F : F \in \mathcal{B}\}) \cap (\bigcap\{F^{\beta:F\in\mathcal{B}\})} \neq \phi$. *Now let* Y *be a member of* $(\cap\{F : F \in \mathcal{B}\}) \cap (\cap\{F^{\beta:F\in\mathcal{B}\})}$ *then* Y *is a* β_1*-accumulation point and* β_2*-accumulation point* $\mathcal{B}$. *The fact that* $\mathcal{B}$ *is a maximal filter base on* Y *implies that* y *is a* β_1*-limit point and* β_2*-limit point of* $\mathcal{B}$.
(d) implies (c) : Let Let B *be any pairwise filter base on* Y *then there exists a pairwise maximal filter base* $\mathcal{B}$ *on* Y *which contains* {B. *By the hypothesis there is a point* Y *such that* Y *is a* β*-limit point and* β_2*-limit point of* $\mathcal{B}$. *Since* $\mathcal{B}$ *is contained in* $\mathcal{B}$, Y *is a* β_1*-accumulation point and* β_2*-accumulation point of* $\mathcal{B}$.
(c) implies (a) : Suppose $\varphi = \{U_\alpha : \alpha \in \gamma\}$ *is* P—*open cover of* Y *with no finite subcover. We clime that the family* $\mathcal{B} = \{Y \sim (\cup_{\alpha\in B} U_\alpha) : \lambda$ = *a finite subset of* γ, $U_\alpha \in \varphi\}$ *is a pairwise filter base on* Y. *Since* φ *is a* P—*open cover of* Y, *their are non void member* U *and* V *of* φ *such that* U *is a* β_1*-open proper subset of* Y *and* V *is* β_2*-open proper subset of* Y. *Hence* Y ∼ U *is a* β_1*-closed proper subset and* Y ∼ V *is a* β_2*-closed proper subset of* Y.

From (d) we have a point Y *in* X *such that* Y *is a* β_1*-accumulation point and* β_2*-accumulation point of* $\mathcal{B}$. *Since* φ *is a pairwise open cover of* X, *there exists a member* V *of* φ *which is a* β_1*-open or* β_2*-open say* β_1*-open and contains* Y *then* $V \cap (X \sim V) = \phi$ *which is contradict to* Y= β_1*-accumulation point of* $\mathcal{B}$.*In (c) the fact that the conditions of filter and net are equivalent can be found easily.*

Corollary 3.2. *If a filter base on a toplogical space (* χ, β_1, β_1 *) has at least one* β_1*-accumulation point and* β_2*-accumulation point, then (* χ, β_1, β_2 *) is a* P—*compact.*

Example 3.1. *Let* Y = $\{y \in F : y \geqslant 0\}$ *and let* β_1 *be discrete topology on* Y *and* β_2 *be indiscreet topology in* Y *then (* χ, β_1, β_2*) is a* P − compact.Let{ *(0,1*$\frac{}{m):m=\text{positive integer}\}}$ *then* $\mathcal{B}$ *is a filter base and every point of* Y *is a* β_2*-accumulation of* $\mathcal{B}$. *On the other hand, any point of* Y *is not a* β_1*-accumulation point of* $\mathcal{B}$.

Definition 3.6. *Let* Y= (χ, β_1, β_2) *be a bitop0logical space,* Y *we say that* P—*countably compact if every countable pairwise open cover has a finite subcover.*

Definition 3.7. *A sequence* $\{b_n\}$ *in* X *we say that pairwise sequence if* $\{b_n\}$ *is not dense in* X *relative to* β_1 *and* β_2.

Definition 3.8. *Let* $Y = (\chi, \beta_1, \beta_2)$ *be a bitoplogical space,* $a \subset Y$ *is called pairwise infinite set in* y *if* a *is an infinite set and is not dense set in* y *relative to* β_1 *and* β_2.

Corollary 3.3. *Let $Y = (\chi, \beta_1, \beta_2)$ be a bitoplogical space, c is sequence in y has at least one β_2-accumulation point then y is pairwise countable compact.*

Example 3.2. *Let Y be the nonnegative real line, and β_1 be the usual topology and $\beta_2 = \{\phi, V \cup (Y, \infty) : V \in \beta_1, x \in y\}$ then (χ, β_1, β_2) is pairwise Hausdorff and pairwise compact. Therefore (χ, β_1, β_2) is a pairwise countably compact. Take a sequence $\{Y_m : Y_m = m$ for each $m \in M\}$ then every point of Y is β_2-accumulation point but any point of Y is not β_1-accumulation point of $\{Y_n\}$.*

Corollary 3.4. *Let $Y = (\chi, \beta_1, \beta_2)$ be a pairwise countably compact space, then Y is pairwise Hausdorff if and only if a sequence in Y has a β_1-limit point and β_2-limit point.*

4. locally countably compact in bitobological space

Definition 4.1. *Let $Y = (\chi, \beta_1, \beta_2)$ be a bitoplogical space, Stoltenberg defined Y as a pairwise locally countably compact if for each $y \in Y$ there is β_1-open neighborhood V of y such that V^{β_1} is β_j-compact $(i \neq j,\ i, j = 1, 2)$ and showed that if (χ, β_1, β_2) is pairwise Hausdorff and pairwise locally countably compact in the sense of Stoltenberg then $\beta_= \beta_2$.*

Example 4.1. *Let $Y = M \cup M_i$ where $i^2 = -1$, M is the set of natural number. Let β_1 be topological generated by $(M - E) \cup G$ where E is a finite subset of M and G is an arbitrary subset of M_i and β_2 is topological generated by $H \cup (_i - E_i)$ where E is a Finite subset of M and H is an arbitrary subset of M. Then (χ, β_1, β_2) is pairwise Hausdorff and pairwise countably compact and (χ, β_1, β_2) is not pairwise locally countably compact defined by Stoltenberg.*

Definition 4.2. *If (χ, β_1, β_2) is a bitoplogical space, then β_1 is locally compact with respect to β_2 if each point of X has a β_1 open neighbourhood whose β_2 Closure is pairwise countably compact.*

Definition 4.3. *A bitoplogical space (χ, β_1, β_2) is pairwise locally countably compact if β_1 is locally compact with respect to β_2 and β_2 is locally compact with respect to β_1 .*

Definition 4.4. *A bitoplogical space (χ, β_1, β_2) is called pairwise locally countably compact if for all $y \in Y$, there is β_i-open neighborhood ν of Y such that $V^{-}j$ is pairwise compact $(i \neq j\ , i, j\ =\ 1, 2)$*

Example 4.2. *Let Y be the real line and β_1 be the usual topology for Y and $\beta_2 = \{\phi\} \cup \{\nu \cup (a, b) : \nu \in \beta_1\}$ then it is easy to see that*

(i) $\beta_1 \neq \beta_2$

(ii) (χ, β_1, β_2) *is not pairwise countably compact, and* (χ, β_1, β_2) *is pairwise locally countably compact.*

Theorem 4.1. *If Y is finite, then the toplogical space (χ, β_1, β_2) is pairwise locally countably compact.*

Proof: Asumme that $Y = \{y_1, y_2, y_3, ..., y_n\}$. Let $y \in Y$ and ν_y be a β_1-open set or a β_2-open set such that $y \in \nu_y$ without less of generalization assume $\nu_y \in \beta_1$, then $y \in \nu_y$, $\overline{\nu_y}$ is finite, so $\overline{\nu_y}$ is pairwise locally countably compact.

Theorem 4.2. *Every pairwise countably compact space is a pairwise locally countably compact. The converse need not be true.*

Example 4.3. *Let $Y = \mathcal{B}$, then ($\mathcal{B}$, β_1, β_{coc}) is a pairwise locally countably compact, but not pairwise countably compact.*
To see this; let $\nu = \{\{A_y\} : y \in Q\} \cup \{Q^c\ ;\ Q^{\ c} \subset \beta_{coc}\}$ be an pairwise open cover. If ν has a finite subcover, then $\mathcal{B} \subset Q^{\ c} \cup \{\{y_1\}, \{y_2\}, \{y_3\}, ..., \{y_n\}\} : Y_i \subset Q\ ; \{i = 1,2,3,...n\}$ which is imopssible.

Theorem 4.3. *Let $X = (\chi, \beta_1, \beta_2)$ be a pairwise Hausdorff toplogical space, then Y is pairwise locally countably compact if and only if for each $y \in Y$ and each β_i-open neighborhood ν of Y, there is a β_i-open neighborhood V of y such that $y \in V \subset V^{\beta \subset U}$ and V^{β} is pairwise compact.*

proof 2. *If* $(X, \mathfrak{T}_1, \mathfrak{T}_2)$ *pairwise locally countably compact. For* $x \in X$, *there is* $\mathfrak{T}_1$*-open* W *such that* $x \in W \subset W_j^{\mathfrak{T}}$, $W_j^{\mathfrak{T}}$ *is pairwise countably compact. By* $W_j^{\mathfrak{T}}$ *is pairwise regular. Let* U *be any* $\mathfrak{T}_i$*-open neighborhood of* x, *then* $W_j^{\mathfrak{T}} \cap U$ *is* $\mathfrak{T}_i$*-open neighborhood of* x *in* $W_j^{\mathfrak{T}}$. *There is* $\mathfrak{T}_i$*-open neighborhood* G *of* x *in* X *such that* $x \in G \cap W^{\mathfrak{T}} \subset (G \cap W^{\mathfrak{T}})^{-\mathfrak{T}} \cap W^{\mathfrak{T}} \subset W^{\mathfrak{T}} \cap U$. *Let* $V = G \cap W$ *then* V *is* $\mathfrak{T}_i$*-open neighborhood of* x *and* $V^{\mathfrak{T}} \subset W^{\mathfrak{T}} \cap U \subset U$. *Since* $V^{\mathfrak{T}}$ *is* $\mathfrak{T}_j$*-closed in pairwise countably compact* $W^{\mathfrak{T}}$, $V^{\mathfrak{T}}$ *is pairwise countably compact.*

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