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Solving the Transportation Problem Using a Multi-Attribute Model

Basher. F. Mohammed [a,*]

[a] Madenat alelem college, Iraq

E-Mail: basher_a@mauc.edu.iq

Abstract

The transportation problem, in operations research, is perhaps one of the issues that is studied more. There exists a plenty number of models to resolve the problem of simply one product, which does not take place now like the identical case of multiples products.

The case had been studied where in assimilating the multiple product problem to the one of a single product. The research has been focused in a proper exposition of the trouble of a couple of transport, and the simplest has cautioned some techniques of solution, for unique cases, that they permit a few simplifications, Located in this example, the goal of this investigation is to provide a model of strategy to the multiple product transportation problem, in which it isn't limited simply to unique cases, but with the intention to be successful to clear up the problem thinking about different factors and uses the multi-attribute technique.

Keywords: Transportation problem, multiple transportation, multi-attribute, decision making.
2010 MSC: 90B06.

1. Introduction

Although in business, it is possible that the problem of transporting multiple products is more frequently encountered, in all textbooks and specialized literature on transport models, the problem of a single product is referred to, and very rarely the case of multiple products is mentioned.

For this reason, many entrepreneurs and their advisors from the point of view of mathematical models, who have to work with the transport of multiple products, must make simplifications, such as considering all their products as one, which in many cases does not have to be done. There is no practical sense.

For this reason, the concern to investigate this topic has arisen, with the ultimate goal of achieving a simple solution that can be put into practice. In this aspect, some progress has already been made, such as the formal approach to the problem and offering some solutions, especially when some simplification is allowed, which can turn the problem into a particular case [1],[2],[3],[4].

But there is still no solution that is capable of solving the problem at any time and considering the presence of different factors that may affect the solution.

On the other hand, given that there is some experience with handling multi-attribute models, and their simplicity is known, the possibility of solving this interesting problem using multi-attribute techniques was raised, thus defining the objective of this research, which can be stated as follows:

*Corresponding author
Email addresses: basher_a@mauc.edu.iq (Basher. F. Mohammed).
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Creating a model to solve the problem of transporting multiple products, considering the presence of different factors that may affect the solution and making use of the multi-attribute technique

With the statement of the general objective, the specific objectives of the research are implicit, which are: to carry out the approach of the transport model of multiple products, including within the relevant variables of other factors, in addition to the different types of products themselves, that can affect its solution, and analyze the multi-attribute models, to finally, using them and solving the problem posed.

The methodology to achieve this objective will be the scientific method applied to operations research, where the first thing is to define the problem, as stated in the objectives, which have just been presented. Next, the data search will be done, to establish the criteria and attributes to be used. This will lead to defining alternatives, which will consist of looking for weightings and value scales that can help solve the problem of transporting multiple products.

Following what is established in the objectives, the last stages will be completed, which are: evaluating the alternatives and selecting the best one, to later present it in its details and analyze its possible solutions, which would be equivalent to establishing controls [5],[6],[7].

Regarding the limitations, in a certain way they were already established when presenting the objectives since a model with a practical sense is desired, which facilitates, for those who have to work with the transport of multiple products, the management of the problem, which helps them in their decision making, although different factors affect it, and that makes use of the multi-attribute technique. But knowing that the main function of the multi-attribute technique is the ordering of the variables, in this work, the second phase will not be considered, that of assigning the numbers of products that each of said variables would be handled.

Additionally, as is the premise of the research group, the product to be obtained must be simple, so that it is not necessary to be an expert in mathematical models to be handle. [8],[9].

2. The multi-product transportation model

When talking about the transport model, we will only try to establish a common language, in terms of nomenclature and, of course, we will start from the reference that represents the transport of a single product, whose scheme is presented in figure (1).

In this figure 1, only three sources have been represented, which are capable of producing, respectively, f1, f2 and f3 units of the product under study, and two destinations that need or demand d1 and d2 units of the same product.

Although each author uses their respective nomenclature, the scheme presented is similar to that found in the vast literature on this subject [1],[2],[3],[4],[5],[6],[7],[8],[9], which, as you can observe, is data for several decades, without major changes.

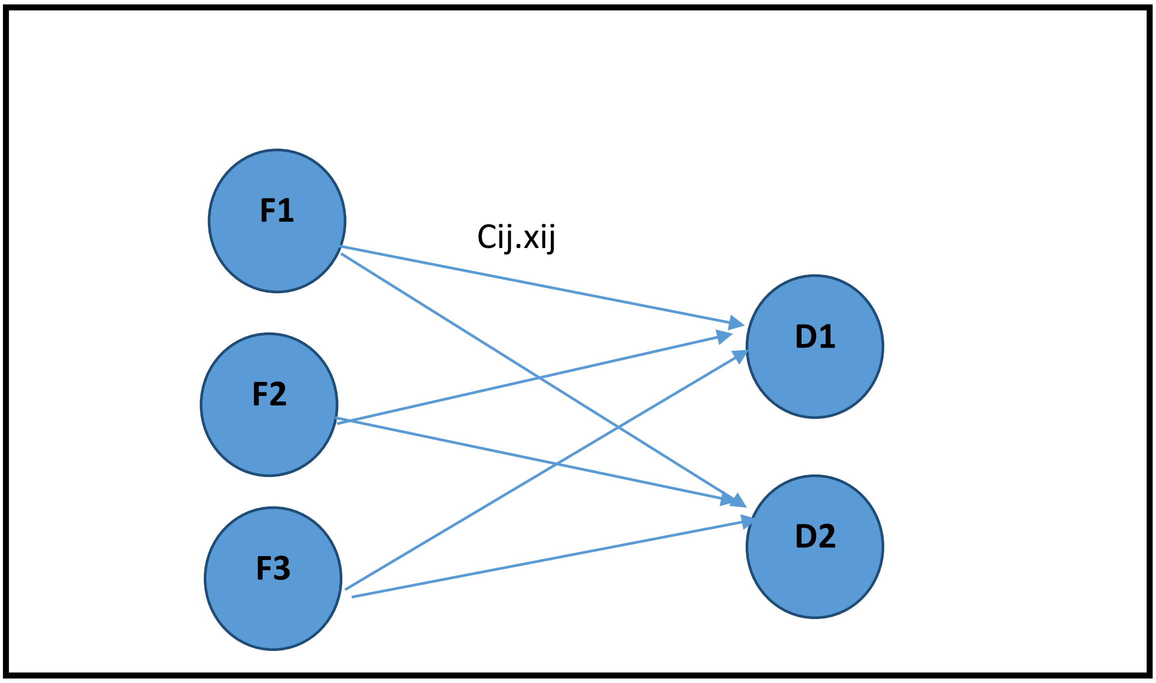

Figure (1): simplified graphical of a transportation problem with three sources and two destinations

The scheme of figure 1 implies that there are six (3*2) variables ($X_{11}, X_{12}, X_{21}, X_{22}, X_{31}, X_{32}$) and each one of them with an associated cost (C11, C12, C21, C22, C31, C32). These variables and these costs have been represented in a generic way, through Xij and cij, which would be read as the quantity of merchandise that is dispatched from source i to destination j, and unit cost of the merchandise that goes from source i to destination j.

Having said the above, the general transport model is deduced for a single product, which aims to minimize costs. Resulting for n sources and m destinations [10],[11],[12]:

$$\text{Min } z = \sum Cij*Xij \quad (1)$$

S.To

$$Xij = fi \quad \ldots\ldots\ldots\; i=1,\ldots,n \quad j=1,\ldots,m \quad (2)$$

$$Xij = dj \quad \ldots\ldots\ldots\; i=1,\ldots,n \quad j=1,\ldots,m \quad (3)$$

$$fi = dj \quad \ldots\ldots\ldots\; i=1,\ldots,n \quad j=1,\ldots,m \quad (4)$$

$$Xij >= 0$$

For all i and all j

Equation one (1), expresses the objective of minimizing the total costs of transporting the merchandise from the different sources to the different destinations. Equations (2) and (3) represent.

the balance in the sources and destinations, respectively, that is, from a source comes what is produced, and to a destiny arrives what is demanded.

Equation (4) represents that the problem is balanced, that is, the total that is produced in the sources is equal to everything demanded in the destinations. And expression (5), simply guarantees that all the variables have a real physical meaning, since they are all positive.

As can be seen from the model, the transportation problem is a linear programming problem, and therefore it can be solved through the Simplex tables. But, given that a sparse matrix is generated, that is, dense in zeros, but also dense in ones, it is not efficient to solve the problem of transporting a single product with the Simplex, and for this reason a large number of problems have arisen. There are methods that allow to solve this problem in an optimal way [13],[14].

Considering the previously presented model, it is more efficient to represent the problem of transporting a single product through the transport model table, as illustrated in Table (1).

In this Table, a first row is considered to identify the destinations, and their respective demands, dj, and each of the following rows will represent the sources and their production capacity, fi, which are shown in the first column. In the central squares of the matrix, each galley will be formed by two elements: Xij and cij, that is, the amount of merchandise that leaves source i to destination j, and its unit cost [10],[11],[12],[13],[14]

Table (1): Transport problems for a product

	d1		d2		....d		m
f1		C11		C12	...		C1m
	X11		X12			X1m	
f2		C21		C22	....		C2m
	X21		X22			X2m	
...	...		....		...	....	
f3		Cn1		Cn2	...		Cnm
	Xn1		Xn2			Xnm	

As, when solving the problem, there will be n * m variables, where only m + n – 1 of them will be basic variables, the methods to solve the problem are two-phase methods, but since they are widely discussed in the literature [2],[3],[4],[5],[7],[9],[11],[12], they will not be commented on in this work, and next to review the model of multiple products.

Making an analogy with the scheme of figure 1, figure 2 is shown, where the problem of multiple products is represented.

Figure (2) shows that the sources (1, 2, ..., i, ..., n), which can independently send the products a, b, ..., k, to be received by the different destinations (1, 2, ..., j, ..., m) [13][14].

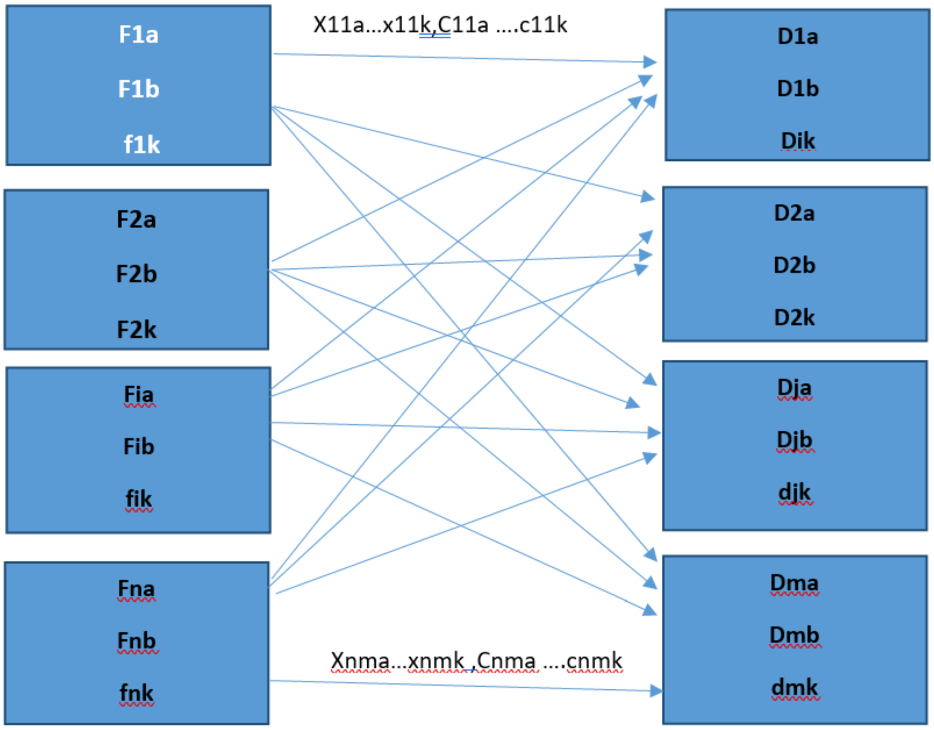

Figure (2): simplified scheme for the problem of transporting multiple products

The multiple transport problem would be reduced to calculating the total minimum cost, under which all destinations could be satisfied in each of the products demanded, according to what they would receive from each source.

And again, making an analogy with the problem of transporting a single product, and as presented in previous works [14],[15],[16],[17], the general model would be reached:

$$\text{Min } z = \sum Cijk*Xijk \quad (6)$$

S.To

$$Xijk = fik \quad .... \; i=1,...,n \quad j=1,...,m \quad (7)$$

with k = a, ..., K

$$Xijk = djk \quad .... \; i=1,...,n \quad j=1,...,m \quad (8)$$

with k = a, ..., K, for i = 1,n

$$fik = djk \quad .... \; i=1,...,n \quad j=1,...,m \quad (9)$$

with k = a, ..., K, for i = 1,n

$$Xijk >= 0 \quad (10)$$

For all i and all j for all k

In this case, as illustrated in Figure 2, there are K products: a, b, ..., K, and generically, from the n sources i, the K products k will be sent to the m destinations j . This allows the structure of the transport model of a single product to be maintained, except that in equation (6), of the costs, the variables have an additional subscript k, which represents the different products, thus as the unit costs that will now be cijk, representing the cost of shipping product k, from source i to destination j.

With respect to equations (7) and (8), each of them now represents a larger grouping of equations, thus (7) represents n * K equations, which would be the multiplication of the K equations produced by each of the products k, by the n equations when repeating the same situation in the n sources i.

And Eq. 8, would represent m * K equations, again the multiplication of the K equations that produces each one of the products k, but now, for each one of the m destinations j.

Similarly, equation (9) represents K equations since it would be the balance for each of the k products independently. As for equation (10), except for the new subscript k, it continues to have the same meaning, of keeping the different variables positive.

This model can be taken to a transport table, very similar to the one for the transport of a single product, presented in Table I, where the space had been intentionally conserved, which will now be used to visualize the different products, both in the sources and in the destinations, as shown in Table (2).

Here, on each of the columns, the demands dja, djb, ..., djk have been placed, and likewise with the sources, in each row the production capacity of the respective source has been placed in each of the fia, fib..., fik products.

In the body of the matrix, a single unit cost and a single variable do not appear, but for each galley, there will be a set of variables Xija, Xijb, ..., Xijk, and their associated costs cija, cijb, .. ., cijk [10],[11],[14].

Table (2): The transportation problem for multiple products

		Destinations							
		D1		D2		......	Dm		supply for products
		Required quantity	costs	Required quantity	costs		Required quantity	costs	
distribution centers or warehouses	F1	x11a	c11a	x12a	c12a	......	x1ma	c1ma	f1a
		x11b	c11b	x12b	c12b	......	x1mb	c1mb	f1b
		x11k	c11k	x12k	c12k	......	x1mk	c1mk	f1k
	F2	x12a	c12a	x22a	c22a	......	x2ma	c2ma	f2a
		x12b	c12b	x22b	c22b	......	x2mb	c2mb	f2b
		x12k	c12k	x22k	c22k	......	x2mk	c2mk	f2k
		......	......	......	......	......	......	......	......
	F3	xn1a	cn1a	xn2a	cn2a	......	xnma	cnma	fna
		xn1b	cn1b	xn2b	cn2b	......	xnmb	cnmb	fnb
		xn1k	cn1k	xn2k	cn2k	......	xnmk	cnmk	fnk
	demand for products	d1a		d2a		......	dma		
		d1b		d2b		......	dmb		
		d1k		d2k		......	dmk		

Now, although the structures are equivalent, it is evident, given that there is not a function that relates the different costs, nor a function that relates the different variables of a galley, that the problem of transporting multiple products cannot be solved, with the same algorithms used to solve the single-product problem.

However, if the details of the model expressed by the equations (6 to 9) are observed, it will be noted that it is still a linear programming problem, for which it must be possible to solve it, with the use of the Simplex method. Which, as was said for a single product, could be inefficient.

This perceived inefficiency of the Simplex method inspires the search for other methods that could allow a good solution, as is the case of multi-attribute models, which are very simple to handle, and could allow a first approximation to the solution of a problem of certain complexity such as the transport of multiple products [12],[13],[14].

The multi-attribute model with multiplicative factors:

In previous works [15],[16],[17],[18], multi-attribute models or multi-attribute utility models (MAU) have been defined as those that are designed to obtain the utility of alternatives through valuable attributes, which must be evaluated as components of the criteria.

In this way, to build a multi-attribute model, it is necessary to: identify relevant criteria and restrictions, list the relevant attributes, perform the weighting of the criteria, determine the proportional weighting of the attributes, determine the scale or measurement range for each attribute, identify possible constraints and apply the MAU model to the feasible alternatives [15],[16] In any case, the final result will be an addition model:

$$pts = i\ Pci * (pajci * vajci) \quad (11)$$

Where the subscript i represents the criterion and the subscript j the attribute, therefore pci will be the score assigned to criterion i, pajci will be the score to attribute j of criterion i, vajci will correspond to the value assigned to attribute j of criterion i, and Pts will be the total value reached by the variable under study.

Due to their way of operating, multi-attribute models are very useful when choosing between different alternatives, or when they must be prioritized. However, what is their greatest strength, additive, which makes them very easy to operate, becomes their main weakness.

This weakness that manifests itself when there are different evaluation scales, or values in very distant ranges, can be corrected through the multiplicative factors, which transform the model into [17],[18].:

$$Pts = kfgk * (i\ fi * pci * (j\ \ pajci * vajci)) \quad (12)$$

That maintains all the previous variables in addition to the use of the multiplicative factors fgk and fi, where k accounts for the number of correction factors, which operate for the entire model, which will be called general factors, the fgk, and fi would represent the factor of correction that operates for criterion i.

These multiplicative factors, normalized between zero and one, which can be continuous, between 0 and 1, or discrete, that is, 0 or 1, give greater flexibility to the multi-attribute model, which with this correction ceases to be a purely additive model [17],[18].

Multi-attribute model of the multiple transport problem

Knowing that the multi-attribute model will be handled through criteria, attributes, weights and values, each of these factors must be located in the multiple transport problem [15],[16].

In the case of criteria and attributes, just as multiplicative factors are usually normalized to zero one, that is, the total sum of the weights of the criteria, as well as the sum of the weights of the attributes of a criterion, must add up to one [15], [17].

On the other hand, the values of each variable to be evaluated, although other scales can be used, in this case they will be assigned between zero and five, which is one of the most frequently used scales [17][18].

As for the model itself, among the main criteria that can be taken into consideration are: products, warehouses, costs, demands, customers, and offers. Since customers could be managed through demands and multiplicative factors, only five criteria were left for the model: Product, Source, Warehouse, Destination, and Cost [15][17][18].

For each of these a criterion factor was created, and additionally two general factors were created. In this way, for product the attributes were handled Weight that.

it supports, Volume that it occupies and Profit that it generates, and the factor, importance which is a continuous factor, which represents how important the product is for the company. Being one (1), it will have maximum importance, and the closer it is to zero (0), its importance will be less [15][16].

For Source, the attributes are: Global Capacity, referred to the source, and Capacity to produce the product, also referred to the source, and the criterion factor, Interesting, is related to the lesser or greater interest in producing that respective product in that specific font, and would go from zero (0) if you don't want to use the font, to one (1), when you prefer to use it. Regarding Warehouse, its attributes are similar to those of Source: Global Capacity, referred to the respective warehouse, and Capacity for the product, also in the respective warehouse, in this case the criterion factor, Conditions, is a continuous factor, where zero (0) represents that there are no conditions to handle the product and the closer to one (1), that these conditions are ideal.

Regarding Destination, the managed attributes are: Demand for the product, Global customer demand and Global importance of the customer, in this case the criterion factor, No demand, will be discrete, with a value of zero (0), when there is no demand of the product and one (1) in any other case [18][19].

Finally, the Cost criterion has the following attributes: Of production, of storage and transport from i to j, i being the source and j the destination. In this case, the factor of the criterion, Total, would go from zero (0), when the total cost, or one of the three lines is very onerous, and will approach one (1), as none of these occur two cases.

This table shows the two general factors, Non-product, whose meaning is that the respective product cannot be produced in that source, in which case its value will be zero (0), and will approach one (1), as it can be produced

Without any difficulty. And the other general factor, Requirement, which is a discrete factor, which depends on the client, and is related to the source, and would take the value zero (0), if you do not want the product to come from that source, it would be worth zero five (0.5) if there is another preferential or specific source, it would take the value zero eight (0.8) if it is one of the preferential sources and it would take the value one (1) if it is a specific source [18][19].

Table (3): Model criteria and attributes

Criteria	Attributes
Product	Weight weight it supports Volume it occupies profit generated
Font	overall capacity Capacity to produce the product
Warehouse	overall capacity product capacity
Destiny	Demand for the product Global customer demand Overall customer importance
Cost	Of production storage Transport from i to j

Table (4): Criterion factors and general factors.

Factor	Guy	Symbol	Meaning	Range
Importance	Criterion Product	f11	Importance for the company	Continuous 0 a 1
Not Interesting	source criteria	f21	Interest in producing the product in that source	Continuous 0 a 1
Conditions	Warehouse criteria	f31	The conditions for storing that product	Continuous 0 a 1
not sue	Destination Criteria	f41	Whether or not the destination demands the product	Discreet 0 to 1
Total	Cost criterion	f51	If a partial cost or the total cost is excessive	Continuous 0 a 1
not product	Factor general	fg1	Possibilities of obtaining the product at the source	Continuous 0 a 1
Requirement	Factor general	(fg2)	Customer preference for the respective source	Discreet 0, 0.5, 0.8 or 1

To finish illustrating the model and visualize its operation, in table V, the criteria and attributes are illustrated, with their respective weights and the fictitious evaluation is made, for one of the variables Xiajk, which would represent a product k, which comes out from store a, from source i, and to destination j.

It is evident that for the evaluation of the different variables, experts in the respective organization must be used, who can handle the importance of each of the criteria, factors and attributes of the model. As

well as the relationship of the respective variables with the same, aspect, however, easy to carry out, due to the highly flexible nature of the model.

As for the values assigned to the Xiajk variables, as can be seen in table V, there are direct ones, a higher valuation is obtained as the value of the attribute is higher, and indirect ones, which would be the opposite case.

In the direct ones, there are: Weight supported, Profit generated, Global Capacity –both at the source and in the warehouse–, Capacity to produce the product, Capacity for the product, Demand for the product, Global customer demand and Global importance of the product and client. For The indirect ones: It occupies Weight, Volume and the costs: of production, of storage and of transport from i to j. [15],[18],[19].

Table (5): A multi-attribute model for the transfer of multiple products

Criteria	Weight	attributes	Weight	Xiajk
Product	0.23	Weight	26%	4
		weight it supports	28%	2
		Volume it occupies	24%	4
		profit generated	22%	4
Importance	f_{11}		90%	
Font	0.19	overall capacity	25%	3
		Capacity to produce the product	75%	4
Interesting	f_{21}		1.00	
Warehouse		overall capacity	20%	4
		product capacity	80%	5
Conditions	f_{31}		1.00	
Destiny	0.27	Demand for the product	35%	5
		Global customer demand	10%	3
		Overall customer importance	55%	3
No demand	$f_{41.}$		1.00	
Cost.		Of production	30%	4
		storage	20%	4
		Transport from i to j	50%	5
total.	$f_{51.}$		1.00	
not product.	$f_{g1.}$		1.00	
Requirement.	$f_{g2.}$		1.00	
			pts	3.867

As can be seen in this table 5, for each variable to be evaluated, a score (Pts) will be obtained, which would be the objective of the multi-attribute model, and which would allow all the variables to be prioritized. After this ordering, the assignment of the products (k), from each warehouse (a), corresponding to the source (i), to the destination (j), and although this assignment is beyond the scope of this work, it can be Summarized as that it will be done by assigning to the variable that is at the head of the list, the maximum possible quantity, which will be given by the minimum, between the remaining demand of the destination and the current availability of the warehouse, at the source.

The solution of the problem will be found when there are no variables to assign, or there are no products in the sources, or when all the destinations have satisfied their demands, which in the case of the problem being balanced, will occur simultaneously.

After the presentation of the model, and having explained its operation, we will now offer the conclusions and recommendations [17],[18],[19].

3. Conclusions and Recommendations

The first conclusion is related to the objectives, which were fully achieved, since a model could be presented, which based on the multi-attribute technique, allows solving the problem of transporting multiple products.

Likewise, it can be concluded that the problem of transporting multiple products is very complex, since it encompasses a large number of variables, which are not only related to the quantity or variety of products that an organization handles from its different sources, but also that they could be linked to factors such as storage capacity and conditions, the demands are specific to the different clients, the degree of importance of these clients and even of the products.

Another aspect to highlight is the breadth of fields of application offered by multi-attribute models, where it can be seen that through a simple model, it can help to solve an extremely complex problem, such as the transport of multiple products.

Special mention deserves the use of multiplicative factors within multi-attribute models, which allows to eliminate alternatives, not valid in a direct way while giving greater flexibility to the models.

In the particular case of the developed model, the multiplicative factors of the criteria: Importance, controls the production of a product, as well as Interesting, it does it for the corresponding source, similarly, Conditions does it for the respective store, favoring or not its Warehousing, No demand, allows a product not to be sent to a customer who did not demand it , and Total, would avoid sending a product from a source to a certain destination, if its costs are not attractive. In the same way, the general multiplicative factors: Non-product and

Requirement, would define whether or not a product is produced and whether or not it is sent to a specific customer.

Although a hypothetical evaluation of the model developed in this work was presented, which allows us to see its applicability, a deeper evaluation of it should be recommended, applying it to a real case if possible.

The comments in these last two paragraphs suggest that much remains to be done with multi-attribute models and multiplicative factors, which is why their use is recommended in other fields, where it is necessary to prioritize variables or conditions.

And finally, it would be recommended to continue research on the problem of transporting multiple products, with the aim of improving the model found, first adapting it to situations of daily business events, and on the other hand, completing the assignment of the amount of merchandise from each source to each destination and also analyze that other variables and restrictions can be incorporated and managed simultaneously.

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