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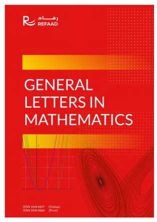

Soft Pre Separation Axioms and Function with Soft Pre Closed Graph

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Abstract

Several notions on soft topology are studied and their basic properties are investigated by using the concept of soft pre open sets and soft pre closure operator which are derived from the basics of soft set theory established by Molodtsov [1]. In this paper we introduce some soft separation axioms called Soft pre R_0 and soft pre R_1 in soft topological spaces which are defined over an initial universe with a fixed set of parameters. Many characterizations and properties of these spaces have been demonstrated. Necessary and sufficient conditions for a soft topological space to be a soft pre R_i space for $i = 0, 1$ were also presented. Furthermore, the concept of functions with soft pre closed graph and soft pre cluster set are defined. Many results on these two concepts are proved. Also, it is proved that a function has a soft pre closed graph if and only if its soft pre cluster set is degenerate.

Keywords: Soft pre open set, soft T_1 space, soft –pre R_i for $i = 0, 1$, Soft pre closure, soft pre kernel.
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1. Introduction

The study of soft sets and their properties was initiated by Molodtsov [1]. Many researchers have followed him, after his introduction of soft set theory as a common mathematical application in dealing with the vagueness of not well defined objects. Several researchers have been applying it on formal modeling, reasoning and computing such as, Shabir and Naz [2], they have described the soft topological spaces and its basic notations in detail. In several papers mathematicians gave many different and interesting topological concepts such as, connectedness [3], compactness [4], separation axioms [5] so on, and they have been extended in soft topological spaces. This paper aims to introduce and give a detailed study to known kinds of separation axioms, by using notations of soft pre open sets and soft pre closure operator.

2. Preliminaries

Throughout the present paper, X will be a nonempty initial universal set and E will be a set of parameters and A be a non-empty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping

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$F : A \rightarrow P(X)$. The collection of soft sets (F, A) over a universe X and the parameter set A is a family of soft sets denoted by $SP(X)_A$.
Here are some definitions and results required in the sequel which can be found in [3], [5], [6], [7] , [8] and [9].

Definition 2.1. For two soft sets (F, A) and (G, B) over a common universe X, we say that (F, A) is a soft subset of (G, B), if 1. $A \subseteq B$ and
2. for all $e \in A$, $F(e) \subseteq G(e)$. We write $(F, A) \sqsubseteq (G, B)$.

Definition 2.2. The complement of a soft set (F, A) is denoted by $(F, A)^c$ or $\tilde{X} \setminus (F, A)$ and is defined by $(F, A)^c = (F^c , A)$ where $F^c : A \rightarrow P(X)$ is a mapping given by $F^c (e) = X \setminus F(e)$, for all $e \in A$.

Definition 2.3. A soft set (F, A) over X is said to be an empty soft set denoted by $\tilde{\phi}$ if for all $e \in A$, $F(e) = \phi$ and (F, A) over X is said to be absolute soft set denoted by A if for all $e \in A$, $F(e) = X$.

Theorem 2.4. *The union of two soft sets of* (F, A) and (G, B) *over the common universe* X *is the soft set* $(H, C) = (F, A) \sqcup (G, B)$, *where* $C = A \cup B$ *and,* $H(e) = \begin{cases} F(e) : \text{if } e \in A - B \\ G(e) : \text{if } e \in B - A \\ F(e) \cup G(e) : \text{if } e \in A \cap B \end{cases}$ *for all* $e \in C$.
In particular, $(F, A) \sqcup (G, A) = (H, A)$ *where* $H(e) = F(e) \cup G(e)$ *for all* $e \in A$.

Definition 2.5. The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe X, denoted $(F, A) \sqcap (G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$. In particular, $(F, A) \sqcap (G, A) = (H, A)$ where $H(e) = F(e) \cap G(e)$ for all $e \in A$.

Definition 2.6. Let $x \in X$, then (x, E) denotes the soft set over X for which $x(e) = \{x\}$, for all $e \in E$. Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$.

Definition 2.7. The soft set (F, E) is called a soft point, denoted by (x_e , E) or x_e , if for the element $e \in E$, $F(e) = \{x\}$ and $F(e) = \varphi$ for all $e \in E \setminus \{e\}$. We say that x_e $\in$ (G, E) if $x \in G(e)$. Two soft points x_e and $y_{e'}$ are distinct if either $\neq y$ or $e \neq e'$. It is clear that $x_e \in (x, E)$ always.

Definition 2.8. [2] Let τ be a collection of soft sets over a universe X with a fixed set A of parameters, then $\tau \subseteq SP(X)_A$ is called a soft topology on X with a fixed set A if,

1. $\tilde{\phi}$, $\tilde{X}$ belong to τ.
2. The union of any number of soft sets in τ belongs to τ.
3. The intersection of any two soft sets in τ belongs to τ.

The triplet (X, A, τ) is called a soft topological space over X. The members of τ are called soft open sets in $\tilde{X}$ and complements of them are called soft closed sets in $\tilde{X}$. Soft operations are denoted by usual set theoretical operations with '∼' symbol above. Soft interior and soft closure are denoted by $\tilde{s}int$ and $\tilde{s}cl$ respectively.

Theorem 2.9. *[6] Arbitrary union of soft open sets is soft open and finite intersection of soft closed sets is soft closed.*

Definition 2.10. [3] Let (X, A, τ) be a soft topological space and let (G, A) be a soft set. Then

1. $\tilde{s}cl(G, A) = \cap\{(K, A) : (K, A)$ is soft closed and $(G, A) \subseteq (K, A)\}$
2. $\tilde{s}int(G, A) = \cup\{(H, A) : (H, A)$ is soft open and $(H, A) \subseteq (G, A)\}$

Definition 2.11. [3] Let (X, τ, A) be a soft topological space over $\tilde{X}$, and (G, A) be a soft set over $\tilde{X}$ and $x_{ff} \tilde{\in} \tilde{X}$. Then (G, A) is said to be a soft neighborhood of x_{ff} if there exists a soft open set (H, A) such that $x_{ff} \tilde{\in} (H, A) \subseteq ((G, A)$.

Definition 2.12. [10] Let (X, τ, A) and (Y, μ, A) be two soft topological spaces. A soft mapping $f_{pu} : \tilde{X} \to \tilde{Y}$ is called soft pre continuous if $f_{pu}^{-1}((G, A)) \in PO(X, \tau, A)$ for all $(G, A) \in PO(Y, \mu, A)$.

Definition 2.13. [11] A soft filter $\mathcal{F}$ converges to a soft point $e_F \in \tilde{X}$ in a soft topological space (X, τ, A) , if every soft neighborhood of the soft point e_F belongs to the soft filter $\mathcal{F}$. It can be denoted by $\mathcal{F} \to e_F$.

Definition 2.14. [12] $\mathcal{F}$ be a soft filter in a soft topological spaces (X, τ, A), a soft point e_F is called soft accumulation point of $\mathcal{F}$, if $e_F \in \tilde{s}cl(G, A)$ for any $(G, A) \in \mathcal{F}$.

Theorem 2.15. *[4] A soft filter F converges to a soft point x_e , then x_e is the soft accumulation point of F , if F is a maximal soft filter and x_e is a soft accumulation point of F , then the soft filter F converges to the soft point x_e .*

Theorem 2.16. *[13] Let $f_{pu} : SP(X)_A \to SP(Y)_B$ be a soft function. If F is a soft ultra filter in X, then ${f_{pu}}^{-1}(F)$ is a soft ultra filter in Y.*

Definition 2.17. [5] A soft topological space $(X,\ \tau,\ A)$ is said to be:

1. Soft pre T_o , if for each pair of distinct soft points x_e , $y_{e'} \in SP(X)_A$, there exist soft pre open sets $(F,\ A)$ and $(G,\ A)$ such that either $x_e \in\ (F,\ A)$ and $y_{e'} \notin (F,\ A)$ or $y_{e'} \in (G,\ A)$ and $x_e \notin (G,\ A)$.
2. Soft pre T_1, if for each pair of distinct soft points x_e , $y_e \in SP(X)_A$, there exist two soft pre open sets (F, A) and (G, A) such that $x_e \in$ (F, A) but $y_e \notin (F,\ A)$ and $y_e \in (G,\ A)$ but $x_e \notin (G,\ A)$.
3. Soft pre T_2, if for each pair of distinct soft points x_e , $y_e \in SP(X)_A$, there exist two disjoint soft pre open sets $(F,\ A)$ and $(G,\ A)$ containing x_e and $y_{e'}$ respectively

Remark 2.18.
1. Every soft T_1-space is a soft T_0-space.
2. Every soft T_2-space is a soft T_1-space

Theorem 2.19. *[3] A space $\tilde{X}$ is soft pre T_1 if and only if each soft point is soft pre closed.*

Theorem 2.20. *[8] A mapping $f : X \to Y$ is soft pre irresolute mapping if and only if the inverse image of every soft pre open set in Y is soft pre open set in X.*

3. Soft Pre R_0 and soft pre R_1 Spases

In this section we define two new types of spaces called soft pre R_i spaces for $i = 0, 1$.

Definition 3.1. A soft topological space (X, τ, A) is called soft pre R_0 if for every soft pre open set (F, A), $\tilde{s}pcl(\{x_\alpha\}) \subseteq (F, A)$ for every $x_\alpha \in (F, A)$.

Definition 3.2. Let (X, τ, A) be a soft topological space and $(F, A) \subseteq \tilde{X}$, then soft pre kernel of (F, A) is defined to be the intersection of all pre open sets containing (F, A) and denoted by $\tilde{s}pker(F, A)$ that is $\tilde{s}pker(F, A) = \cap\{(G, A) \in \tilde{s}pO(X), (F, A) \subseteq (G, A)\}$.

Lemma 3.3. *Let (X, τ, A) be a soft topological space, and $x_\alpha \in \tilde{X}$ then $y_{\alpha^\circ} \in \tilde{s}pker(\{x_\alpha\})$ if and only if $x_\alpha \in \tilde{s}pcl(\{y_{\alpha^\circ}\})$.*

Proof. Suppose that $x_\alpha \notin \tilde{s}pker(F, A)$, then there exists a soft pre open set (F, A) containing x_α such that $y_{\alpha^\circ} \notin (F, A)$. Therefore, we have $x_\alpha \notin \tilde{s}pcl(\{y_{\alpha^\circ}\})$. The proof of converse case can be done similarly. □

Theorem 3.4. *For a soft topological space (X, τ, A) the following properties are equivalent:*

1. (X, τ, A) *is* $\tilde{s}pR_0$.
2. *For any $(K, A) \in \tilde{s}pC(X)$ and $x_\alpha \notin (K, A)$ there exists $(F, A) \in \tilde{s}pO(X)$ such that $(K, A) \subseteq (F, A)$ and $x_\alpha \notin (F, A)$.*
3. *For any $(K, A) \in \tilde{s}pC(X)$ and $x_\alpha \notin (K, A)$ implies that $(K, A) \cap \tilde{s}pcl(\{x_\alpha\}) = \widetilde{\emptyset}$.*

4. *For any distinct soft points* $x_\alpha, y_{\alpha^\circ} \in \tilde{X}$ *either* $\tilde{s}pcl(\{x_\alpha\}) = \tilde{s}pcl(\{y_{\alpha^\circ}\})$ *or* $\tilde{s}pcl(\{x_\alpha\}) \cap \tilde{s}pcl(\{y_{\alpha^\circ}\}) = \widetilde{\phi}$.

Proof. $1 \to 2$. Let $(K, A) \in \tilde{s}pC(X)$and $x_\alpha \notin (K, A)$. Then by 1 $\tilde{s}pcl(\{x_\alpha\}) \subseteq \tilde{X} \diagdown (K, A)$, let $(F, A) = \tilde{X} \diagdown (K, A)$, then $(F, A) \in \tilde{s}pO(X)$, $(K, A) \subseteq (F, A)$ and $x_\alpha \notin (F, A)$.
$2 \to 3$. Let $(K, A) \in \tilde{s}pC(X)$ and $x_\alpha \notin (K, A)$.Then there exists $(F, A) \in \tilde{s}pO(X)$ such that $(K, A) \subseteq (F, A)$ and $x_\alpha \notin (F, A)$.Since $(K, A) \subseteq (F, A)$, so by 2 $(F, A) \cap \tilde{s}pcl(\{x_\alpha\}) = \widetilde{\emptyset}$, this implies that $(K, A) \cap \tilde{s}pcl(\{x_\alpha\}) = \widetilde{\emptyset}$
$3 \to 4$. Let x_α and y_{α° be two distinct soft points of $\tilde{X}$. Suppose that $\tilde{s}pcl(\{x_\alpha\}) \neq \tilde{s}pcl(\{y_{\alpha^\circ}\})$, then there exist a soft point z_α such that $z_\alpha \in \tilde{s}pcl(\{x_\alpha\})$ and $z_\alpha \notin \tilde{s}pcl(\{y_{\alpha^\circ}\})$ [or $z_\alpha \in \tilde{s}pcl(\{y_{\alpha^\circ}\})$ such that $z_\alpha \notin \tilde{s}pcl(\{x_\alpha\})$] and there exists $(F, A) \in \tilde{s}pO(X)$ such that $y_{\alpha^\circ} \notin (F, A)$ and $z_\alpha \in (F, A)$, hence $x_\alpha \in (F, A)$, therefore , we have $x_\alpha \notin \tilde{s}pcl(\{y_{\alpha^\circ}\})$ by 3 we obtain $\tilde{s}pcl(\{x_\alpha\}) \cap \tilde{s}pcl(\{y_{\alpha^\circ}\}) = \widetilde{\emptyset}$.
$4 \to 1$. Let $F, A) \in \tilde{s}pO(X)$ and $x_\alpha \in (F, A)$, for each $y_{\alpha^\circ} \notin (F, A)$. Then $x_\alpha \neq y_{\alpha^\circ}$ and $x_\alpha \notin \tilde{s}pcl(\{y_{\alpha^\circ}\})$, this shows that $\tilde{s}pcl(\{x_\alpha\}) \neq \tilde{s}pcl(\{y_{\alpha^\circ}\})$, by 4 we have $\tilde{s}pcl(\{x_\alpha\}) \cap \tilde{s}pcl(\{y_{\alpha^\circ}\}) = \widetilde{\emptyset}$, for each $y_{\alpha^\circ} \in \tilde{X} \diagdown (F, A)$. On the other hand, since $(F, A) \in \tilde{s}pO(X)$and $y_{\alpha^\circ} \in \tilde{X} \diagdown (F, A)$, we have $\tilde{s}pcl(\{y_{\alpha^\circ}\}) \subseteq \tilde{X} \diagdown (F, A)$. Hence $\tilde{X} \diagdown (F, A) = \cup \tilde{s}pcl(\{y_{\alpha^\circ}\})$ where $y_{\alpha^\circ} \in \tilde{X} \diagdown (F, A)$. Therefore, we obtain that $\tilde{X} \diagdown (F, A) \cap \tilde{s}pcl(\{x_\alpha\}) = \widetilde{\emptyset}$ and $\tilde{s}pcl(\{x_\alpha\}) \subseteq (F, A)$. This shows that (X, τ, A) is soft pre R_0. □

Theorem 3.5. *A topological space* (X, τ, A) *is a soft pre* R_0 *space if and only if for any* x_α *and* y_{α° *in* $\tilde{X}$, $\tilde{s}pcl(\{x_\alpha\}) \neq \tilde{s}pcl(\{y_{\alpha^\circ}\})$ *implies* $\tilde{s}pcl(\{x_\alpha\}) \cap \tilde{s}pcl(\{y_{\alpha^\circ}\}) = \widetilde{\phi}$

Proof. This is an immediate consequence of Theorem 3.4. □

Lemma 3.6. *The following statements are equivalent for any distinct soft points* x_α and y_{α° *in a soft topological space* (X, τ, A).

1. $\tilde{s}pker(\{x_\alpha\}) \neq \tilde{s}pker(\{y_{\alpha^\circ}\})$,
2. $\tilde{s}pcl(\{x_\alpha\}) \neq \tilde{s}pcl(\{y_\alpha\})$.

Proof. $1 \to 2$: Suppose that $\tilde{s}pker(\{x_\alpha\}) \neq \tilde{s}pker(\{y_{\alpha^\circ}\})$. Then there exists a soft point $z_\alpha \in \tilde{X}$ such that $z_\alpha \in \tilde{s}pker(\{x_\alpha\})$ and $z_\alpha \notin \tilde{s}pker(\{y_{\alpha^\circ}\})$. Since $z_\alpha \in \tilde{s}pker(\{x_\alpha\})$ so $\{x_\alpha\} \cap \tilde{s}pcl(\{z_\alpha\}) \neq \widetilde{\emptyset}$. This implies that $x_\alpha \in \tilde{s}pcl(\{z_\alpha\})$ and since $z_\alpha \notin \tilde{s}pker(\{y_{\alpha^\circ}\})$ we have $\{y_{\alpha^\circ}\} \cap \tilde{s}pcl(\{z_\alpha\}) = \widetilde{\emptyset}$. Since $x_\alpha \in \tilde{s}pcl(\{z_\alpha\})$, so $\tilde{s}pcl(\{x_\alpha\}) \subseteq \tilde{s}pcl(\{z_\alpha\})$ and hence $\{y_{\alpha^\circ}\} \cap \tilde{s}pcl(\{x_\alpha\}) = \widetilde{\emptyset}$. Therefore, $\tilde{s}pcl(\{x_\alpha\}) \neq \tilde{s}pcl(\{y_{\alpha^\circ}\})$.
$2 \to 1$: Suppose that $\tilde{s}pcl(\{x_\alpha\}) \neq \tilde{s}pcl(\{y_{\alpha^\circ}\})$. Then there exists a soft point z_α in $\tilde{X}$ such that $z_\alpha \in \tilde{s}pcl(\{x_\alpha\})$ and $z_\alpha \notin \tilde{s}pcl(\{y_{\alpha^\circ}\})$ there exists an soft pre open set (F,A) containing z_α (and hence x_α) but not y_{α°, that is $y_{\alpha^\circ} \notin \tilde{s}pker(\{x_\alpha\})$. Therefore, $\tilde{s}pker(\{x_\alpha\}) \neq \tilde{s}pker(\{y_{\alpha^\circ}\})$. □

Theorem 3.7. *A soft topological space* (X, τ, A) *is soft pre* R_0 *if and only if for any two distinct soft points* $x_\alpha, y_{\alpha^\circ} \in \tilde{X}$, $\tilde{s}pker(\{x_\alpha\}) \neq \tilde{s}pker(\{y_{\alpha^\circ}\})$ *implies* $\tilde{s}pker(\{x_\alpha\}) \cap \tilde{s}pker(\{y_{\alpha^\circ}\}) = \widetilde{\phi}$.

Proof. Necessity, suppose that (X, τ, A) is soft pre R_0. Thus by Lemma 3.6, for any distinct soft point x_α and y_{α° in $\tilde{X}$, if $\tilde{s}pker(\{x_\alpha\}) \neq \tilde{s}pker(\{y_{\alpha^\circ}\})$, then $\tilde{s}pcl(\{x_\alpha\}) \neq \tilde{s}pcl(\{y_{\alpha^\circ}\})$. Assume that $z_\alpha \in \tilde{s}pker(\{x_\alpha\}) \cap \tilde{s}pker(\{y_{\alpha^\circ}\})$, since $z_\alpha \in \tilde{s}pker(\{x_\alpha\})$, and Lemma 3.4 it follows that $x_\alpha \in \tilde{s}pcl(\{z_\alpha\})$. Since $x_\alpha \in \tilde{s}pcl(\{x_\alpha\})$, by Theorem 3.4 $\tilde{s}pcl(\{x_\alpha\}) = \tilde{s}pcl(\{z_\alpha\})$. Similarly we have $\tilde{s}pcl(\{x_\alpha\}) = \tilde{s}pcl(\{z_\alpha\}) = \tilde{s}pcl(\{y_{\alpha^\circ}\})$, which is contradiction. Therefore, $\tilde{s}pker(\{x_\alpha\}) \cap \tilde{s}pker(\{y_{\alpha^\circ}\}) = \widetilde{\phi}$.
Sufficiency, let (X, τ, A) be a soft topological space such that for any distinct soft points x_α and y_{α° in $\tilde{X}$, $\tilde{s}pker(\{x_\alpha\}) \neq \tilde{s}pker(\{y_{\alpha^\circ}\})$ implies that, $\tilde{s}pker(\{x_\alpha\}) \cap \tilde{s}pker(\{y_\alpha\}) = \widetilde{\phi}$. If $\tilde{s}pcl(\{x_\alpha\}) \neq \tilde{s}pcl(\{y_{\alpha^\circ}\})$, hence by Lemma 3.6, $\tilde{s}pker(\{x_\alpha\}) \neq \tilde{s}pker(\{y_{\alpha^\circ}\})$. Therefore, $\tilde{s}pker(\{x_\alpha\}) \cap \tilde{s}pker(\{y_{\alpha^\circ}\}) = \widetilde{\phi}$. Which implies that $\tilde{s}pcl(\{x_\alpha\}) \cap \tilde{s}pcl(\{y_{\alpha^\circ}\}) = \widetilde{\phi}$, because $z_\alpha \in \tilde{s}pcl(\{x_\alpha\})$ implies that $x_\alpha \in \tilde{s}pker(\{z_\alpha\})$ so $\tilde{s}pker(\{x_\alpha\}) \cap \tilde{s}pker(\{y_{\alpha^\circ}\}) \neq \widetilde{\phi}$. By hypothesis we have , $\tilde{s}pker(\{x_\alpha\}) = \tilde{s}pker(\{z_\alpha\})$, then $z_\alpha \in \tilde{s}pcl(\{x_\alpha\}) \cap \tilde{s}pcl(\{y_{\alpha^\circ}\})$ implies that $\tilde{s}pker(\{x_\alpha\}) = \tilde{s}pker(\{z_\alpha\}) = \tilde{s}pker(\{y_{\alpha^\circ}\})$, this is contradiction. Therefore, $\tilde{s}pcl(\{x_\alpha\}) \cap \tilde{s}pcl(\{y_{\alpha^\circ}\}) = \widetilde{\phi}$. Hence by Theorem 3.4, (X, τ, A) is soft pre R_0. □

Theorem 3.8. *For a soft topological space* (X, τ, A), *the following statements are equivalent:*

1. (X, τ, A) *is soft pre* R_0.
2. *For any non-empty soft set (F,A) and* $(G, A) \in \tilde{s}pO(X)$ *such that* $(F, A) \cap (G, A) \neq \widetilde{\phi}$, *there exists* $(K, A) \in \tilde{s}pC(X)$ *such that* $(F, A) \cap (K, A) \neq \widetilde{\phi}$ *and* $(K, A) \subseteq (G, A)$.
3. *For any* $(G, A) \in \tilde{s}pO(X)$, $(G, A) = \cup\{(K, A) \in \check{s}pC(X); (K, A) \subseteq (G, A)\}$
4. *For any* $(K, A) \in \check{s}pC(X)$, $(K, A) = \cap\{(G, A) \in \tilde{s}pO(X); (K, A) \subseteq (G, A)\}$
5. *For any* $x_\alpha \in \tilde{X}$, $\tilde{s}pcl(\{x_\alpha\}) \subseteq \tilde{s}pker(\{x_\alpha\})$.

Proof. 1→2: Let (F, A) be a non empty subset of $\tilde{X}$ and $(G, A) \in \tilde{s}pO(X)$ such that $(F, A) \cap (G, A) \neq \widetilde{\phi}$. Let $x_\alpha \in (F, A) \cap (G, A)$. Since $x_\alpha \in (G, A) \subseteq \tilde{s}pO(X)$, so by 1, we have $\tilde{s}pcl(\{x_\alpha\}) \subseteq (G, A)$. Set $(K, A) = \tilde{s}pcl(\{x_\alpha\})$ then $(K, A) \in \check{s}pC(X)$ such that $(K, A) \subseteq (G, A)$. and $(F, A) \cap (K, A) \neq \widetilde{\phi}$.
2→3 : Let $(G, A) \in \tilde{s}pO(X)$. Then $\cup\{(K, A) \in \check{s}pC(X); (K, A) \subseteq (G, A)\}$. Now let x_α be any soft point of (G, A). By 2 there exists $(K, A) \in \check{s}pC(X)$, such that $x_\alpha \in (K, A)$ and $(K, A) \subseteq (G, A)$. Therefore, we have $x_\alpha \in (K, A) \subseteq \cup\{(K, A) \in \check{s}pC(X); (K, A) \subseteq (G, A)\}$. Hence $(G, A) = \cup\{(K, A) \in \check{s}pC(X); (K, A) \subseteq (G, A)\}$.
3→4 : Obvious.
4→5 : Let x_α be any soft point of $\tilde{X}$ and $y_{\alpha^\circ} \notin \tilde{s}pker(\{x_\alpha\})$. So there exists $(H, A) \in \tilde{s}pO(X)$ such that $x_\alpha \in (H, A)$ and $y_{\alpha^\circ} \notin (H, A)$. Hence $\tilde{s}pcl(\{y_\alpha\}) \cap (H, A) = \widetilde{\phi}$. By 4, we have $[\cap\{(G, A) \in \tilde{S}O(X); \tilde{s}pcl(\{y_{\alpha^\circ}\}) \subseteq (G, A)\}] \cap (H, A) = \widetilde{\phi}$, where $(G, A) \in \tilde{s}pO(X)$ such that $x_\alpha \notin (G, A)$ and $\tilde{s}pcl(\{y_{\alpha^\circ}\}) \subseteq (G, A)$. Therefore $\tilde{s}pcl(\{x_\alpha\}) \cap (G, A) = \widetilde{\phi}$ and hence $y_\alpha \notin \tilde{s}pcl(\{x_\alpha\})$. Consequently, we obtain that $\tilde{s}pcl(\{x_\alpha\}) \subseteq \tilde{s}pker(\{x_\alpha\})$.
4→5 : Let $(G, A) \in \tilde{s}pO(X)$ and $x_\alpha \in (G, A)$, let $y_{\alpha^\circ} \in \tilde{s}pker(\{x_\alpha\})$. Then $x_\alpha \in \tilde{s}pcl(\{y_{\alpha^\circ}\})$ and $y_{\alpha^\circ} \in (G, A)$ this implies that $\tilde{s}pker(\{x_\alpha\}) \subseteq (G, A)$. Therefore, we obtain $\tilde{s}pcl(\{x_\alpha\}) \subseteq \tilde{s}pker(\{x_\alpha\}) \subseteq (G, A)$. This shows that (X, τ, A) is soft pre R_0. □

Theorem 3.9. *For a soft topological space* (X, τ, A), *the following statements are equivalent :*

1. (X, τ, A) *is a soft pre* R_0 *space.*
2. $\tilde{s}pcl(\{x_\alpha\}) = \tilde{s}pker(\{x_\alpha\})$, *for all* $x_\alpha \in \tilde{X}$.

Proof. Suppose that $(X, \emptyset, A)$ is a soft pre R_0 space. By Theorem 3.8 $\widetilde{s}pcl(\{x_\alpha\}) \subseteq \check{s}pker(\{x_\alpha\})$, for all $x_\alpha \widetilde{\in} \widetilde{X}$. Let $y_{\alpha^\circ} \widetilde{\in} \widetilde{s}pker(\{x_\alpha\})$, then $x_\alpha \widetilde{\in} \widetilde{s}pcl(\{y_{\alpha^\circ}\})$, and by Theorem 3.4 $\widetilde{s}pcl(\{x_\alpha\}) \widetilde{=} \widetilde{s}pcl(\{y_{\alpha^\circ}\})$. Therefore $y_{\alpha^\circ} \widetilde{\in} \widetilde{s}pcl(\{x_\alpha\})$ and hence $\widetilde{s}pker(\{x_\alpha\}) \widetilde{\subseteq} \widetilde{s}pcl(\{x_\alpha\})$. This shows that $\widetilde{s}pcl(\{x_\alpha\}) \widetilde{=} \widetilde{s}pker(\{x_\alpha\})$, for all $x_\alpha \widetilde{\in} \widetilde{X}$. 2→1 :Follows from Theorem 3.8. □

Theorem 3.10. *For a soft topological space* (X, τ, A), *the following statements are equivalent :*

1. (X, τ, A) *is a soft pre* R_0 *space.*
2. $x_\alpha \in \tilde{s}pcl(\{y_{\alpha^\circ}\})$ *if and only if* $y_{\alpha^\circ} \in \tilde{s}pcl(\{x_\alpha\})$ *for any two distinct soft points* $x_\alpha, y_{\alpha^\circ} \in \tilde{X}$.

Proof. 1→2 : Assume that (X, τ, A) is a soft pre R_0 space. Let $x_\alpha \in \tilde{s}pcl(\{y_{\alpha^\circ}\})$ and (H, A) be any soft pre open set containing y_{α°. By 1 $\tilde{s}pcl(\{y_{\alpha^\circ}\}) \subseteq (H, A)$, hence $x_\alpha \in (H, A)$. Therefore, every soft pre open set containing y_{α° contains x_α, so $y_{\alpha^\circ} \in \tilde{s}pcl(\{x_\alpha\})$.
2→1 : Let (G, A) be any soft pre open set containing x_α, if $y_{\alpha^\circ} \notin (G, A)$, then $x_\alpha \notin \tilde{s}pcl(\{y_{\alpha^\circ}\})$ and By 2, we have $y_{\alpha^\circ} \notin \tilde{s}pcl(\{x_\alpha\})$. This implies that $\tilde{s}pcl(\{x_\alpha\}) \subseteq (G, A)$, hence (X, τ, A) is a soft pre R_0 space. □

Theorem 3.11. *A soft topological space* (X, τ, A) *is a soft pre* R_0 *if and only if* $\tilde{s}pker(\{x_\alpha\}) \neq \tilde{s}pker(\{y_{\alpha^\circ}\})$ *, for all* $x_\alpha \neq y_{\alpha^\circ}$

Proof. Follows from Theorem 3.9 and Theorem 3.10. □

Lemma 3.12. *Let* (X, τ, A) *be a soft topological space and* $(F, A) \subseteq \tilde{X}$.
Then $\tilde{s}pker(F, A) = \left\{x_\alpha \in \tilde{X} | \tilde{s}pcl(\{x_\alpha\}) \cap (F, A) \neq \widetilde{\emptyset}\right\}$.

Proof. Let $x_\alpha \in \tilde{s}pker(F,A)$ and suppose $\tilde{s}pcl(\{x_\alpha\}) \cap (F,A) = \widetilde{\emptyset}$.
Hence $x_\alpha \notin \tilde{X} \backslash \tilde{s}pcl(\{x_\alpha\})$ which is a soft pre open set containing (F,A) and this is impossible, since $x_\alpha \in \tilde{s}pker(F,A)$, hence $\tilde{s}pcl(\{x_\alpha\}) \cap (F,A) \neq \widetilde{\emptyset}$. A gain let $x_\alpha \in \tilde{X}$ such that $\tilde{s}pcl(\{x_\alpha\}) \cap (F,A) \neq \widetilde{\emptyset}$ and suppose that $x_\alpha \notin \tilde{s}pker(F,A)$. Then there exists a soft pre open (G,A), such that $x_\alpha \notin (G,A)$ and $(F,A) \subseteq (G,A)$. Let $y_{\alpha^\circ} \in \tilde{s}pcl(\{x_\alpha\}) \cap (F,A)$. Hence (G,A) is a soft pre neigbourhood of y_α which does not contain x_α. This contradict that $x_\alpha \in \tilde{s}pker(F,A)$ so the claim. □

Theorem 3.13. *For a soft topological space (X,τ,A), the following statements are equivalent :*

1. (X,τ,A) *is a soft pre* R_0 *space.*
2. *If* (K,A) *is soft pre closed, then* $(K,A) = \tilde{s}pker(K,A)$.
3. *If* (K,A) *is soft pre closed, and* $x_\alpha \in (K,A)$, *then* $\tilde{s}pker(\{x_\alpha\}) \subseteq (K,A)$.
4. *If* $x_\alpha \in \tilde{X}$, *then* $\tilde{s}pker(\{x_\alpha\}) \subseteq \tilde{s}pcl(\{x_\alpha\})$.

Proof. $1 \to 2$: Let (K,A) be a soft pre closed set and $x_\alpha \notin (K,A)$. Thus $\tilde{X} \setminus (K,A)$ is soft pre open set containing x_α. Since $\tilde{X}$ is an soft pre R_0 space, so $\tilde{s}pcl(\{x_\alpha\}) \subseteq \tilde{X} \setminus (K,A)$, thus $\tilde{s}pcl(\{x_\alpha\}) \cap (K,A) = \widetilde{\phi}$, by Lemma 3.12 $x_\alpha \notin \tilde{s}pker(K,A)$. Therefore $\tilde{s}pker(\{x_\alpha\}) \subseteq (K,A)$, hence $(K,A) = \tilde{s}pker(K,A)$.
$2 \to 3$: In general $(F,A) \subseteq (G,A)$ implies that $\tilde{s}pker(F,A) \subseteq \tilde{s}pker(G,A)$. Therefore, it follows from 2 that $\tilde{s}pker(\{x_\alpha\}) \subseteq \tilde{s}pker(K,A) = (K,A)$.
$3 \to 4$: Since $x_\alpha \in \tilde{s}pcl(\{x_\alpha\})$ and $\tilde{s}pcl(\{x_\alpha\})$ is soft closed, so by 3, we get that $\tilde{s}pker(\{x_\alpha\}) \subseteq \tilde{s}pcl(\{x_\alpha\})$.
$4 \to 1$: Let $x_\alpha \in \tilde{s}pcl(\{y_{\alpha^\circ}\})$, then by Lemma 3.3 $y_{\alpha^\circ} \in \tilde{s}pker(\{x_\alpha\})$. Since $x_\alpha \in \tilde{s}pcl(\{x_\alpha\})$ and $\tilde{s}pcl(\{x_\alpha\})$ is soft pre closed. So by 4 we obtain $y_{\alpha^\circ} \in \tilde{s}pker(\{x_\alpha\}) \subseteq \tilde{s}pcl(\{x_\alpha\})$. Therefore $x_\alpha \in \tilde{s}pcl(\{y_{\alpha^\circ}\})$ implies that $y_{\alpha^\circ} \in \tilde{s}pcl(\{x_\alpha\})$, on the same way, if $y_{\alpha^\circ} \in \tilde{s}pcl(\{x_\alpha\})$, we get $x_\alpha \in \tilde{s}pcl(\{y_{\alpha^\circ}\})$, so by Theorem 3.10 (X,τ,A) is a soft pre R_0 space. □

Definition 3.14. A soft filter base $\mathcal{F}$ is called soft p-convergent to a point x_α in $\tilde{X}$, if for any soft pre open set (H,A) of $\tilde{X}$ containing x_α there exists (G,A) in $\mathcal{F}$ such that $(G,A) \tilde{\sqsubseteq} (H,A)$.

Lemma 3.15. *Let (X,τ,A) be a soft topological space and let x_α and y_{α° be any two points in $\tilde{X}$ such that every net in $\tilde{X}$ $\tilde{s}p$-converging to y_{α° $\tilde{s}p$ -converges to x_α. Then $x_\alpha \in \tilde{s}pcl(\{y_{\alpha^\circ}\})$.*

Proof. Suppose that $x_{\alpha i} = y_{\alpha^\circ}$ for each $i \in N$. Then $\{x_{\alpha i}\}_{i\in N}$ is a net in $\tilde{s}pcl(\{y_{\alpha^\circ}\})$. By the fact that $\{x_{\alpha i}\}_{i\in N}$ $\tilde{s}p$-converges to y_{α°, then $\{x_{\alpha i}\}_{i\in N}$ $\tilde{s}p$ -converges to x_α and this means that $x_\alpha \in \tilde{s}pcl(\{y_{\alpha^\circ}\})$. □

Theorem 3.16. *For a topological space (X,τ,A), the following statements are equivalent :*
1 (X,τ,A) *is a soft pre* R_0
2*If* $x_\alpha, y_{\alpha^\circ} \in \tilde{X}$, *then* $y_{\alpha^\circ} \in \tilde{s}pcl(\{x_\alpha\})$ *if and only if every net in* $\tilde{X}$ $\tilde{s}p$ *-converging to* y_{α° $\tilde{s}p$ *-converges to* x_α.

Proof. $1 \to 2$: Let $x_\alpha, y_{\alpha^\circ} \in \tilde{X}$ such that $y_{\alpha^\circ} \in \tilde{s}pcl(\{x_\alpha\})$. Let $\{x_{\alpha i}\}_{i\in\wedge}$ be a net in $\tilde{X}$ such that $\{x_{\alpha i}\}_{i\in\wedge}$ $\tilde{s}p$ -converges to y_{α°. Since $y_{\alpha^\circ} \in \tilde{s}pcl(\{x_\alpha\})$, by Theorem 3.5 we have $\tilde{s}pcl(\{x_\alpha\}) = \tilde{s}pcl(\{y_\alpha\})$. Therefore $x_\alpha \in \tilde{s}pcl(\{y_\alpha\})$. This means that $\{x_{\alpha i}\}_{i\in\wedge}$ $\tilde{s}p$ -converges to x_α. Conversely, let $x_\alpha, y_{\alpha^\circ} \in \tilde{X}$ such that every net in $\tilde{X}$ $\tilde{s}p$ -converging to y_{α° $\tilde{s}p$ -converges to x_α. Then $x_\alpha \in \tilde{s}pcl(\{y_{\alpha^\circ}\})$ by Lemma 3.12. By Theorem 3.6, we have $\tilde{s}pcl(\{x_\alpha\}) = \tilde{s}pcl(\{y_{\alpha^\circ}\})$. Therefore $y_{\alpha^\circ} \in \tilde{s}pcl(\{x_\alpha\})$
$2 \to 1$: Assume that x_α and y_{α° are any two points of $\tilde{X}$ such that $\tilde{s}pcl(\{x_\alpha\}) \cap \tilde{s}pcl(\{y_{\alpha^\circ}\}) \neq \widetilde{\phi}$. Let $z_\alpha \in \tilde{s}pcl(\{x_\alpha\}) \cap \tilde{s}pcl(\{y_{\alpha^\circ}\})$. So there exists a net $\{x_{\alpha i}\}_{i\in\wedge}$ in $\tilde{s}pcl(\{x_\alpha\})$ such that $\{x_{\alpha i}\}_{i\in\wedge}$ $\tilde{s}p$ -converges to z_α. Since $z_\alpha \in \tilde{s}pcl(\{y_{\alpha^\circ}\})$ then $\{x_{\alpha i}\}_{i\in\wedge}$ $\tilde{s}p$ -converges to y_{α°. It follows that $y_{\alpha^\circ} \in \tilde{s}pcl(\{x_\alpha\})$. By the similarly we obtain $x_\alpha \in \tilde{s}pcl(\{y_{\alpha^\circ}\})$. Therefore $\tilde{s}pcl(\{x_\alpha\}) = \tilde{s}pcl(\{y_{\alpha^\circ}\})$ and by Theorem 3.6 (X,τ,A) is a soft pre R_0 □

Definition 3.17. A soft topological space (X,τ,A) is called soft pre R_1 if for $x_\alpha, y_{\alpha^\circ} \in \tilde{X}$ with $\tilde{s}pcl(\{x_\alpha\}) \neq \tilde{s}pcl(\{y_{\alpha^\circ}\})$ there exist disjoint soft pre open sets (F,A) and (G,A) such that $\tilde{s}pcl(\{x_\alpha\}) \subseteq (F,A)$ and $\tilde{s}pcl(\{y_{\alpha^\circ}\}) \subseteq (G,A)$.

Theorem 3.18. *If* (X, τ, A) *is soft pre* R_1*, then it is a soft pre* R_0 *space.*

Proof. Suppose that (X, τ, A) is soft pre R_1. Let (H, A) be any soft pre open set containing a soft point x_α. Then for each $y_{\alpha^\circ} \in \tilde{X} \setminus (H, A)$, $\tilde{s}pcl(\{x_\alpha\}) \neq \tilde{s}pcl(\{y_{\alpha^\circ}\})$. Since (X, τ, A) is soft pre R_1, there exist (K, A) and (G, A) such that $\tilde{s}pcl(\{x_\alpha\}) \subseteq (K, A)$ and $\tilde{s}pcl(\{y_{\alpha^\circ}\}) \subseteq (G, A)$. Let $(F, A) = \cup\{(G, A) : y_{\alpha^\circ} \in \tilde{X} \setminus (H, A)\}$, then $\tilde{X} \setminus (H, A) \subseteq (F, A)$, $x_\alpha \notin (F, A)$ and (F, A) is a soft pre open set. Therefore, $\tilde{s}pcl(\{x_\alpha\}) \subseteq \tilde{X} \setminus (F, A) \subseteq (H, A)$. Hence (X, τ, A) is soft pre R_0. □

Proposition 3.19. *Every soft pre* T_1 *space is soft pre* R_0*.*

Proof. Obvious . □

Proposition 3.20. *A soft topological space* (X, τ, A) *is soft pre* T_1 *if and only if* $\tilde{X}$ *is both soft pre* T_0 *and soft pre* R_0*.*

Proof. Necessity, Let $\tilde{X}$ be soft pre T_1, then by Proposition 3.19, $\tilde{X}$ is soft pre R_0 and since every soft pre T_1 is soft pre T_0 that completes the proof.
Sufficiency, assume that $\tilde{X}$ is both soft pre T_0 and soft pre R_0 . Let x_α, $y_{\alpha^\circ} \in \tilde{X}$ be any pair of distinct soft points, since $\tilde{X}$ is soft pre T_0, there exists a soft pre open set (H, A) such that $x_\alpha \in (H, A)$ and $y_{\alpha^\circ} \notin (H, A)$ or there exists a soft pre open set (G, A) such that $y_{\alpha^\circ} \in (G, A)$ and $x_\alpha \notin (G, A)$. Suppose that $x_\alpha \in (H, A)$ and $y_{\alpha^\circ} \notin (H, A)$. Since $\tilde{X}$ is soft pre R_0, then $\tilde{s}pcl(\{x_\alpha\}) \subseteq (H, A)$. As $y_{\alpha^\circ} \notin (H, A)$ implies $y_{\alpha^\circ} \notin \tilde{s}pcl(\{x_\alpha\})$. Hence $y_{\alpha^\circ} \in (G, A) = \tilde{X} \setminus \tilde{s}pcl(\{x_\alpha\})$ and it is clear that $x_\alpha \notin (G, A)$, this implies that there exist soft pre open set (G, A) and (H, A) containing x_α and y_{α° respectively such that $x_\alpha \notin (G, A)$ and $y_{\alpha^\circ} \notin (H, A)$. Therefore (X, τ, A) is a soft pre T_1 space. □

Theorem 3.21. *A space* $\tilde{X}$ *is soft pre* R_0 *if and only if for every soft pre closed set* (K, A) *and* $x_\alpha \notin (K, A)$*, there exists a soft pre open set* (G, A) *such that* $x_\alpha \notin (G, A)$ *and* $(K, A) \subseteq (G, A)$*.*

Proof. Let $\tilde{X}$ be soft pre R_0 space and (K, A) be soft pre closed subset of $\tilde{X}$ not containing $x_\alpha \in X$. Then $X \setminus (K, A)$ is soft pre open set containing x_α, since $\tilde{X}$ is soft pre R_0 space implies that $\tilde{s}pcl(\{x_\alpha\}) \subseteq X \setminus (K, A)$ and then $(K, A) \subseteq X \setminus \tilde{s}pcl(\{x_\alpha\})$. Now let $(G, A) = X \setminus \tilde{s}pcl(\{x_\alpha\})$, then (G, A) is soft pre open set not contains x_α and $(K, A) \subseteq (G, A)$.
Conversely: Let $x_\alpha \in (G, A)$ where (G, A) is soft pre open set in $\tilde{X}$. Then $X \setminus (G, A)$ is soft pre closed set and $x_\alpha \notin X \setminus (G, A)$, by hypothesis there exists a soft pre open set (H, A)such that $x_\alpha \notin (H, A)$ and $X \setminus (G, A) \subseteq (H, A)$. Now $X \setminus (H, A) \subseteq (G, A)$ and $x_\alpha \in X \setminus (H, A)$, but $X \setminus (H, A)$ is soft pre closed set then $\tilde{s}pcl(\{x_\alpha\}) \subseteq X \setminus (H, A) \subseteq (G, A)$ this implies that $\tilde{X}$ is a soft pre R_0 space. □

Theorem 3.22. *A space* $\tilde{X}$ *is soft pre* T_2 *if and only if it is soft pre* R_1 *and soft pre* T_0*.*

Proof. Let $\tilde{X}$ be soft pre T_2. Then from Definition 2.17 $\tilde{X}$ is soft pre T_0 and to show $\tilde{X}$ is soft pre R_1 space, let $x_\alpha, y_{\alpha^\circ} \in \tilde{X}$ such that $\tilde{s}pcl(\{x_\alpha\}) \neq \tilde{s}pcl(\{y_{\alpha^\circ}\})$ and since $\tilde{X}$ is soft pre T_1 space so by Theorem 2.19, every singleton set in $\tilde{X}$ is soft pre closed, this means $\tilde{s}pcl(\{x_\alpha\}) = \{x_\alpha\}$ and $\tilde{s}pcl(\{y_{\alpha^\circ}\}) = \{y_{\alpha^\circ}\}$ implies that $\{x_\alpha\} \neq \{y_{\alpha^\circ}\}$ and since $\tilde{X}$ is soft pre T_2 space so there exist two disjoint soft pre open sets (G, A) and (H, A) such that $x_\alpha \in (G, A)$ and $y_{\alpha^\circ} \in (H, A)$ implies that $\tilde{s}pcl(\{x_\alpha\}) \subseteq (G, A)$ and $\tilde{s}pcl(\{y_{\alpha^\circ}\}) \subseteq (H, A)$. Thus $\tilde{X}$ is a soft pre R_1space.
Conversely, let $\tilde{X}$ be soft pre R_1 and soft pre T_0 space and $x_\alpha, y_{\alpha^\circ} \in \check{X}$ such that $x_\alpha \neq y_{\alpha^\circ}$. Now since $\tilde{X}$ is soft pre T_0 so by Definition 2.17 there exist a soft pre open set (G, A) such that $x_\alpha \in (G, A)$ and $y_{\alpha^\circ} \notin (G, A)$ or $y_\alpha \in (G, A)$ and $x_\alpha \notin (G, A)$, take $x_\alpha \in (G, A)$ and $y_{\alpha^\circ} \notin (G, A)$ implies that $(G, A) \cap \{y_{\alpha^\circ}\} = \phi$, and then $x_\alpha \notin \tilde{s}pcl(\{y_{\alpha^\circ}\})$ this implies that $\tilde{s}pcl(\{x_\alpha\}) \neq \tilde{s}pcl(\{y_{\alpha^\circ}\})$ and since $\tilde{X}$ is soft pre R_1 so there exist two disjoint soft pre open sets (G, A) and (H, A) such that $\tilde{s}pcl(\{x_\alpha\}) \subseteq (G, A)$ and $\tilde{s}pcl(\{y_{\alpha^\circ}\}) \subseteq (H, A)$ implies that $x_\alpha \in (G, A)$ and $y_{\alpha^\circ} \in (H, A)$. Thus $\tilde{X}$ is soft pre T_2. □

4. Functions with soft *pre* closed graph and soft pre-cluster set

In this section we discuss properties of functions defined on soft topological spaces that have soft pre closed graphs. Moreover we investigate properties of soft pre cluster sets and its relation with functions.

Definition 4.1. The graph of a function $f_{pu} : (X, \tau.A) \to (Y, \mu, A)$ is $G(f_{pu})$ and it is soft pre closed in $\tilde{X} \times \tilde{Y}$, if for each $(x_\alpha, y_{\alpha^\circ}) \in (\tilde{X} \times \tilde{Y}) \backslash G(f_{pu})$, there exist two soft pre open sets (U, A) containing x_α and (V, A) containing y_{α° such that $(U, A) \times (V, A) \cap G\,(f_{pu}) = \widetilde{\phi}$.

Lemma 4.2. *The function* $f_{pu} : (X, \tau.A) \to (Y, \mu, A)$ *has a soft pre closed graph if and only if for each* $x_\alpha \in \tilde{X}$ *and* $y_{\alpha^\circ} \in \tilde{Y}$ *such that* $f_{pu}(x_\alpha) \neq y_{\alpha^\circ}$ *, there exist two soft pre open sets* (U, A) *and* (V, A) *containing* x_α *and* y_{α° *respectively, such that* $f_{pu}((U, A) \cap (V, A)) = \widetilde{\phi}$

Proof. Follows from Definition 4.1. □

Proposition 4.3. *If* $f_{pu} : (X, \tau.A) \to (Y, \mu, A)$ *is an injective function with, soft pre closed graph, then* $\tilde{X}$ *is a soft pre* T_0 *space.*

Proof. Let $x_{\alpha 1}$ and $x_{\alpha 2}$ be two distinct points in $\tilde{X}$. Since f_{pu} is injective, so $f_{pu}(x_{\alpha 1}) \neq f_{pu}(x_{\alpha 2})$. Let $f_{pu}\,(x_{\alpha 1}) = y_{\alpha 1}$thus $f_{pu}(x_{\alpha 2}) \neq y_{\alpha 1}$, by Lemma 4.2, there exists two soft pre open sets (U, A) and (V, A) containing $x_{\alpha 2}$ and $y_{\alpha 1}$ respectively, such that $f_{pu}((U, A) \cap (V, A)) = \widetilde{\phi}$, then $(U, A) \cap f_{pu}^{-1}\,(V, A) = \widetilde{\phi}$. We get $f_{pu}\,(x_{\alpha 1}) = y_{\alpha 1} \in (V, A)$, then $x_{\alpha 1} \in f_{pu}^{-1}\,(V, A)$ implies that, $x_{\alpha 1} \notin (U, A)$. Again consider $f_{pu}\,(x_{\alpha 2}) = y_{\alpha 2}$ implies that $f_{pu}(x_{\alpha 1}) \neq y_{\alpha 2}$. Since the graph of f_{pu} is soft pre closed, so there exist soft pre open sets (U_1, A) containing $x_{\alpha 1}$ and (U_1, A) containing $y_{\alpha 2}$ such that $f_{pu}((U_1, A) \cap (V_1, A)) = \widetilde{\phi}$, so $(U_1, A) \cap f_{pu}^{-1}\,(V_1, A) = \widetilde{\phi}$, we obtain $f_{pu}\,(x_{\alpha 2}) = y_{\alpha 2} \in (V_1, A)$, hence $x_{\alpha 2} \in f_{pu}^{-1}\,(V_1, A)$ and hence $x_{\alpha 2} \notin (U_1, A)$. Therefore $\tilde{X}$ is a soft pre T_1 space. □

Proposition 4.4. *If* $f_{pu} : (X, \tau.A) \to (Y, \mu, A)$ *is a surjective function with soft pre closed graph, then* $\tilde{Y}$ *is a soft pre* T_1 *space.*

Proof. Let $y_{\alpha 1}$and $y_{\alpha 2}$ be two distinct points of $\tilde{Y}$. Since f_{pu} is surjective so there exists a soft point $x_{\alpha 1} \in \tilde{X}$, with $f_{pu}\,(x_{\alpha 1}) = y_{\alpha 1}$ then $f_{pu}(x_{\alpha 1}) \neq y_{\alpha 2}$. Therefore, $(x_{\alpha 1}, y_{\alpha 2}) \in G(f_{pu})$, since the graph of f_{pu} is soft pre closed, by Lemma 4.2, there exist two soft pre open sets (U_1, A) in $\tilde{X}$ containing $x_{\alpha 1}$ and (V_2, A) in $\tilde{Y}$ containing $y_{\alpha 2}$ such that $f_{pu}((U_1, A) \cap (V_2, A)) = \widetilde{\phi}$. We obtain $y_{\alpha 2} \in (V_2, A)$, and $x_{\alpha 1} \in (U_1, A)$ implies that $f_{pu}\,(x_{\alpha 1}) \in f_{pu}\,(U_1, A)$, so $y_{\alpha 1} \notin (V_2, A)$. Again from the surjectivity of f_{pu} there exists $x_{\alpha 2} \in \tilde{X}$ with $f_{pu}\,(x_{\alpha 2}) = y_{\alpha 2}$, then $f_{pu}(x_{\alpha 2}) \neq y_{\alpha 1}$, thus $(x_{\alpha 2}, y_{\alpha 1}) \notin G(f_{pu})$, and the graph of f_{pu} is soft pre closed, there exist two soft pre open sets (U_2, A) and (V_1, A) containing $x_{\alpha 2}$ and $y_{\alpha 1}$ respectively, such that $f_{pu}((U_2, A) \cap (V_1, A)) = \widetilde{\phi}$. We get $x_{\alpha 2} \in (U_2, A)$ implies that $y_{\alpha 2} = f_{pu}\,(x_{\alpha 2}) \in f_{pu}\,(U_2, A)$, so $Y_{\alpha 2} \notin (V_1, A)$. It follows that $\tilde{Y}$ is a soft pre T_1-space. □

Lemma 4.5. *If* $f_{pu} : (X, \tau.A) \to (Y, \mu, A)$ *is a bijective function with soft pre closed graph, then both* $\tilde{X}$ *and* $\tilde{Y}$ *are soft pre* T_1 *spaces.*

Proof. Follows from Proposition 4.3 and Proposition 4.4 □

Proposition 4.6. *If* $f_{pu} : (X, \tau.A) \to (Y, \mu, A)$ *is soft pre irresolute mapping and* Y *is a soft pre* T_2*- space, then* $G\,(f_{pu})$ *is soft pre closed.*

Proof. Suppose that $(x_\alpha, y_{\alpha^\circ}) \notin G(f_{pu})$. Then $f_{pu}\,(x_\alpha) \neq y_{\alpha^\circ}$, and since $\tilde{Y}$ is a soft pre T_2- space, there exist soft pre open sets (U, A) and (V, A) such that $f_{pu}\,(x_\alpha) \in (U, A)$, $y_{\alpha^\circ} \in (V, A)$ and $((U, A) \cap (V, A)) = \widetilde{\phi}$. Since f_{pu} is soft pre irresolute mapping, so there exists a soft pre open set (G, A) containing x_α such that $f_{pu}\,(G, A) \subseteq (U, A)$, hence we have $f_{pu}\,(G, A) \cap (V, A) = \widetilde{\phi}$. Therefore, by Lemma 4.2, $G\,(f_{pu})$ is soft pre closed. □

Definition 4.7. Let $f_{pu}:(X,\tau.A)\to(Y,\mu,A)$ be any soft function, the soft pre cluster set of f_{pu} at x_α is denoted by $\tilde{s}pC(f_{pu},x_\alpha)$ is the set of all points $y_{\alpha^\circ}\in\tilde{Y}$ such that whenever there exists a Filter base $\mathcal{F}$ soft-pre converges to a point x_α, the filter base $f_{pu}(\mathcal{F})$ soft pre converges to the point y_{α°.

One of the characterizations of soft pre cluster set of a function f_{pu} is clarified in the following result:

Theorem 4.8. *Let* f_{pu}: $(X,\tau.A)\to(Y,\mu,A)$ *be any function and* $x_\alpha\in\hat{X}$ *so the following statements are equivalent:*

1. $y^\circ_\alpha\in\hat{s}pC(f_{pu},x_\alpha)$
2. $y_\alpha\in\in\prod\{\hat{s}pclf_{pu}((U,A)):$ *for all* $(U,A)\in\hat{s}pN(x_\alpha)\}$.
3. $f_{pu}(\hat{s}pN(x_\alpha))$ *is soft pre accumulates to* y_{α°.
4. $f^{-1}_{pu}(\hat{s}pN(y^\circ_\alpha))$ *is soft pre accumulates to* x_α.
5. $x_\alpha\in\prod\{\hat{s}pclf^{-1}_{pu}((V,A)):$ *for all* $(V,A)\in\hat{s}pN(y_{\alpha^\circ})\}$.

Proof. $1\longrightarrow2$: Let $y_\alpha\in\hat{s}pC(f_{pu},x_\alpha)$ so there exists a net x_α soft pre converges to a point x_α and $f_{pu}(x_\alpha)$ soft pre converges to a point y_α. Suppose that (U,A) is any soft pre open set containing x_α, since $x_{\alpha i}$ soft pre converges to x_α, there exist $i_0\in D$ such that $x_{\alpha i}\in(U,A)$ for each $i\geqslant i_0$ and $f_{pu}(x_\alpha)$ soft pre converges to y_α. Therefore, $y_\alpha\in\hat{s}pclf_{pu}(x_\alpha)$ implies that $y_\alpha\in\hat{s}pclf_{pu}((U,A))$ for each soft pre open set (U,A) containing x_α. Hence $y_\alpha\in\Pi\{\hat{s}pclf_{pu}((U,A)):$ for all $(U,A)\in\hat{s}pN(x_\alpha)\}$.
$2\to3$: Let $y_\alpha\in\Pi\{\hat{s}pclf_{pu}((U,A)):$ forall$(U,A)\in\hat{s}pN(x_\alpha)\}$, so $y_{\alpha^\circ}\in\hat{s}pclf_{pu}((U,A))$ for each soft pre open set (U,A) containing x_α. Then $f_{pu}((U,A))\cap(V,A)\neq\widehat{\phi}$ for each soft pre open sets (U,A) containing x_α and (V,A) containing y_{a° implies that $f_{pu}\left(N_{s_p}(x_\alpha)\right)\cap(V,A)\neq\widehat{\phi}$ for every soft pre open set (V,A) containing $y_\alpha{}^\circ$ and hence $f_{pu}(\widehat{s}pN(x_\alpha))$ is soft pre accumulates to y_α.
$3\to4$: Let $f_{pu}(\widehat{s}pN(x_\alpha))$ is soft pre accumulates to y_{α°, which implies that $f_{pu}(\hat{s}pN(x_\alpha))\Pi(V,A)\neq\widehat{\phi}$ for each $(V,A)\in\hat{s}pN(y_{a^\circ})$, thus $(U,A)\Pi f^{-1}_{pu}(\hat{s}pN(y_{a^2}))\neq\widehat{\phi}$ for every soft pre open set (U,A) in $\hat{X}$ containing x_α it follows that $f^{-1}_{pu}(\hat{s}pN(y_{\alpha^\circ}))$ is soft pre accumulates to x_α.
$4\to5$: Assume that $f^{-1}_{pu}(\hat{s}pN(y_{\alpha^\circ}))$ is soft pre accumulates to x_α, so $(U,A)\Pi f^{-1}_{pu}(\hat{s}pN(y_{\alpha^\circ}))\neq\widehat{\phi}$, for every soft pre open set (U,A) containing x_α. It follows that $(U,A)\Pi f^{-1}_{pu}((V,A))\neq\widehat{\phi}$ for every soft pre open set (U,A) in $\hat{X}$ containing x_α and (V,A) in $\hat{Y}$ containing $y_\alpha{}^\circ$ and hence $x_\alpha\in\hat{s}pclf_{pu}((V,A))$ for every soft pre open set (V,A) containing y_α. This shows that $x_\alpha\in\Pi\left\{\hat{s}pclf^{-1}_{pu}((V,A)):\text{ for all }(V,A)\in\hat{s}pN(y_{\alpha^2})\right\}$.
$5\to1$: Since $\hat{S}O(X,x_\alpha)$ is a filter base which is soft pre converges to a point x_α, then $\hat{S}O(X,x_\alpha)$ is contained in a nutria filter F on $\hat{X}$ which is also soft pre-converges to x_α, so there exist $(F,A)\in\mathcal{F}$ such that $(F,A)\subseteq(U,A)$ for every soft pre open set (U,A) in $\hat{X}$ containing x_α, so $(U,A)\in\mathcal{F}$. By 5 $x_\alpha\in\hat{s}pclf^{-1}_{pu}((V,A))$ for every soft pre open set (V,A) containing y_α. So $(U,A)\cap f^{-1}_{pu}((V,A))\neq\widehat{\phi}$, implies that $f_{pu}((U,A))\cap(V,A)\neq\widehat{\phi}$ for every soft pre open set (U,A) containing x_α and (V,A) containing $y_\alpha{}^\circ$. Hence $y_\alpha{}^\circ$ is soft pre adheres point of ultra filter base $f_{pu}(F)$ and by Corollary 4.5,$f_{pu}(F)$ is soft pre converges to a point y_{α°. □

By using the concept of soft pre cluster set of a function f_{pu}: $(X,\tau.A)\to(Y,\mu,A)$ we obtain some properties and characterizations of the soft pre graph f_{pu}. We start by the following result which is a relation between a function with soft pre closed graph and soft pre cluster set of the function.

Definition 4.9. Let $(X,\tau.A)$ be a soft topological space, the degenerate soft pre cluster set of f_{pu} is a soft pre cluster set which contain exactly one element.

Theorem 4.10. *Let* f_{pu}: $(X,\tau.A)\to(Y,\mu,A)$ *be any function, the graph of* f_{pu} *is soft pre closed if and only if soft pre cluster set of* f_{pu} *at* x_α *is degenerate.*

Proof. Let y_α be any point in $\hat{Y}$ different from $f_{pu}(x_\alpha)$ by Lemma 4.2, there exist $(U,A)\in\hat{s}pO(X,x)$ and $(V,A)\in\hat{s}pO(Y,y)$, such that $f_{pu}((U,A))\Pi(V,A)=\widehat{\phi}$ implies that $y_\alpha\notin\hat{s}pclf_{pu}((U,A))$ and by Theorem 4.9, $y_\alpha\notin\hat{s}pC(f_{pu},x_\alpha)$. Hence $\hat{s}pC(f_{pu},x_\alpha)=\{f_{pu}(x_\alpha)\}$.
Conversely, suppose $G(f_{pu})$ is not soft pre closed. This implies that there exists $(x_\alpha,y_{\alpha^\circ})\notin G(f_{pu})$

such that $f_{pu}((U,A))\sqcap(V,A)\neq\widehat{\phi}$ for every soft pre open set (U,A) in $\hat{X}$ containing x_α and (V,A) in $\hat{Y}$ containing y_α, then $y_\alpha\in\hat{s}pclf_{pu}((U,A))$ for each soft pre open set (U,A) containing x_α by Theorem 4.8, $y_\alpha\in\hat{s}pC\,(f_{pu},x_\alpha)$ which contradicts the fact that $\hat{s}pC\,(f_{pu},x_\alpha)=\{f_{pu}\,(x_\alpha)\}$. Therefore $G\left(f_{pu}\right)$ is soft pre closed □

From above theorem we obtain the following results:

Corollary 4.11. *Let* f_{pu}: $(X,\tau.A)\rightarrow(Y,\mu,A)$ *be any function, the graph of* f_{pu} *is soft pre closed if and only if there exists a net* x_α *soft pre converges to a point* x_α *the net* $f_{pu}\,(x_\alpha)$ *soft pre converges to a point* y_α *and* $y_\alpha\,^\circ=f_{pu}\,(x_\alpha)$.

Corollary 4.12. *Let* f_{pu}: $(X,\tau.A)\rightarrow(Y,\mu,A)$ *be any function, the graph of* f_{pu} *is soft pre closed if and only if there exists a filter* F *soft pre convergls to a point* x_α, *the net* $f_{pu}(\mathcal{F})$ *soft pre converges to a point* y_α *and* $y_\alpha=f_{pu}\,(x_\alpha)$.

Corollary 4.13. *The graph of a function* f_{pu}: $(X,\tau.A)\rightarrow(Y,\mu,A)$ *is soft pre closed if and only if* $f_{pu}\,(x_\alpha)\in\cap\hat{s}pclf_{pu}((U,A))$ *for each soft pre open set* (U,A) *containing* x_α.

5. Conclution

In the last two decades the soft set theory, new definitions, examples, new classes of soft sets, and properties for mappings between different classes of soft sets are introduced and studied. After that, the theory of soft topological spaces is investigated. This paper continues the study of the theory of soft topological spaces. In section 3, we present the notion of soft pre R_i spaces for $i = 0,1$, we get several characterizations and properties of these two spaces. In section 4, we obtain nice results concerning functions with soft closed graphs and its relations with the notion of soft convergence and cluster set of a function.

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