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Abstract:

Capturing the dynamic properties of financial market volatility has always been very auspicious in asset pricing. The mean reverting properties of financial markets are of paramount importance for investment decision makers with far reaching implications. The purpose of this study was to investigate stochastic volatility in international financial markets during periods of financial distress. Accordingly, a Heston model was used to stochastic volatility in the CAC 40, DAX, JSE, Nasdaq Index and the Nikkei-225 during the 2007-2008 financial crisis and the Covid-19 pandemic. This study used January 1, 2020 to December 31, 2021 and December 1, 2007 to June 30, 2009 as the sample period representing the Covid-19 pandemic and financial crisis respectively. The findings of this study revealed the extortionate stochastic volatility during the Covid-19 pandemic compared to the financial crisis. There will be slightly higher than normal arbitrage opportunities and mispricing in most financial markets as a result of the pandemic.

***Keywords**: Stochastic volatility; Heston model; Financial distress; Covid-19 pandemic; Asset pricing.*

الملخص:

لطالما كان الحصول على الخصائص الديناميكية لتقلبات الأسواق المالية أمرًا مبشرًا للغاية في تسعير الأصول. إن متوسط خصائص العودة للأسواق المالية له أهمية قصوى بالنسبة لصناع القرار الاستثماري مع تداعيات بعيدة المدى. فالغرض من هذه الدراسة هو التحقيق في التقلبات العشوائية في الأسواق المالية الدولية خلال فترات الضائقة المالية. وفقًا لذلك، تم استخدام نموذج هيستون للتقلب العشوائي في CAC 40 و DAX و JSE ومؤشر Nasdaq و Nikkei-225 خلال الأزمة المالية 2007-2008 ووباء كورونا. واستخدمت هذه الدراسة 1 يناير 2020 إلى 31 ديسمبر 2021 و 1 ديسمبر 2007 إلى 30 يونيو 2009 كفترة عينة تمثل جائحة كورونا والأزمة المالية على التوالي. فكشفت نتائج هذه الدراسة التقلب العشوائي الباهظ خلال جائحة كورونا مقارنة بالأزمة المالية. وأنه ستكون هناك فرص موازنة وسوء تسعير أعلى قليلًا من المعتاد في معظم الأسواق المالية نتيجة للوباء.

***الكلمات المفتاحية:** تقلب عشوائي؛ نموذج هيستون؛ ضائقة مالية؛ جائحة كورونا؛ تسعير الأصول.*

1. Introduction

A puzzling albeit unpredictable concept in finance literature is the concept of stochastic volatility due to its usual and unsettling nature. While the stylized facts of financial market volatility have been extensively investigated (Nobert & Andrea, 1996; Annaert, De Ceuster & Valckx, 2001; Opschoor, 2014; Gerlach, Ramaswamy & Scatigna,2006), its stochastic nature and implications seems to be an unending controversy. The term stochastic volatility is used to describe the variance of a stochastics process that is randomly distributed (Mastroeni, 2022). The adjective "stochastic" is added to the word volatility to describe the variance of a security index that follows a random process (He & Chen, 2021). In capturing this randomness, the output of a stochastic model quantifies the underlying tendency to revert back to its long term mean and variance values (Maffra, Armstrong & Pennanen, 2021). Also, it assumes a non-constant variance which is not affected by the price of the asset (Moawia, Floros & Gkillas, 2020). In doing so, stochastics models are a leap from traditional autoregressive models such as Black-Scholes model. The quintence of estimating stochastic volatility in stock markets should include a random process controlled by the concept of Brownian motion so as to capture the unobserved properties that will affect the dynamics of asset pricing (Rambharat & Brockwell, 2010). The past few years have been highly eventful leading to heightened volatility in financial markets. Coupled with the pandemic, most central banks around the world have consistently raised interest rates with the Federal Reserve bank in the United States, that is proposing another seventy-five percent basis point increase to curb the rising inflation. The G10 nations are also set to implement a similar action in 2023. These macro-economic policies were also evident in the 2007-2008 financial crisis where central banks raised interest rates due to the then uncertain environment. During periods of financial distress, there is an influx of risk off trading and safe heavens flow due to surging inflation. The spread of recessionary fear causes financial markets to take a hawkish trend. With this heightened uncertainty, unprecedented volatility runs well above the forecasted targets. Notwithstanding the above mentioned, Li and Koopman (2020) had contended that there are unobserved components that drives volatility. Stochastic volatility is as a result of uncertainty, where asset valuation will most certainly be overpriced and market participants need to be compensated for taking additional risks (Posedel Šimović, & Tafro, 2021). There will be a low mean reverting property in markets where stochastic volatility is present, leading to low expectations of future returns (Ke Peng, Xun & Hu, 2021). Therefore, investigating stochastic volatility in financial markets, especially during periods of financial distress is necessary, in other words, to explore the asymptotic behaviour of the conditional distribution of demeaned index returns. This is well possible from the calibrated parameters in the stochastic process that have estimated likelihood, long term variance, drift, correlation and p-values. Therefore, the aim of this study was to investigate stochastic volatility in international financial markets during the Covid-19 pandemic and 2007-2008 financial crisis. The study furthered its analysis to draw a parallel comparison between the Covid-19 pandemic and 2007-2008 financial and its implication to asset pricing. Accordingly, this study seeks to answer the following Questions: is there empirical evidence of stochastic volatility during periods of financial distress? Is there empirical evidence of long run memories and mean reversion in financial markets? Should market participants expect to see arbitrage opportunities in the market? This study contributes not only to the literature of volatility modelling and forecasting during periods of financial distress but also analysis long term stock market pricing dynamics driven by separate strands of random processes.

2. Literature

Black and Scholes (1973) introduced the Black-Scholes model that had a closed form solution with constant volatility. This model depicts that price variations over time in a financial instrument such as stock index. In 1976, the first stochastic volatility model was introduced by Cox & Ross (1976). Cox & Ross (1976) assumed a mixture of continuous and jump processes and a return to volatility relationship that is a deterministic function of the underlying asset. Black (1976) introduced another volatility model which included a leverage effect where returns are compared to a predetermined volatility distribution of an underlying asset. In Black's (1976) model, it was observed that there is a negative correlation between returns and volatility although it was not used for valuation. Engle (1982) also developed an ARCH model to account for the non-constant variance which was coined heteroscedasticity. Bollerslev et al. (1986) extended Engle's (1982) ARCH model by including moving averages and certain parameters to capture both the past price movements and the past estimated volatility. Later on, Hull and White (1987) introduced the first stochastics volatility model which was semi closed with no correlation between returns. On Monday the 9th 1987, the DOW Jones dropped by more than twenty percent within a day which was later called black Monday. Since then, the prevalence of volatility smiles has been evident in option markets which contradicts the Black-Scholes model. Accordingly, Stein and Stein (1991) developed another Stochastics volatility model that used a mean reverting process called Olhstein-Uihbeck process to model the variance of an asset's return. Their original work assumed no correlation between returns and volatility. Heston (1993) introduced the novel stochastic volatility model which incorporated modelling the square root process of the variance of an underlying. Heston's (1993) model was the popular choice because of its easy closed form solution that required numerical integration with no negative variance. Heston's (1993) model also incorporated the leverage effect first identified by Black (1976). Dupire (1994) introduced a local volatility model where market prices were used to determine volatility surfaces used in several option pricing models. The diagram below depicts the model.

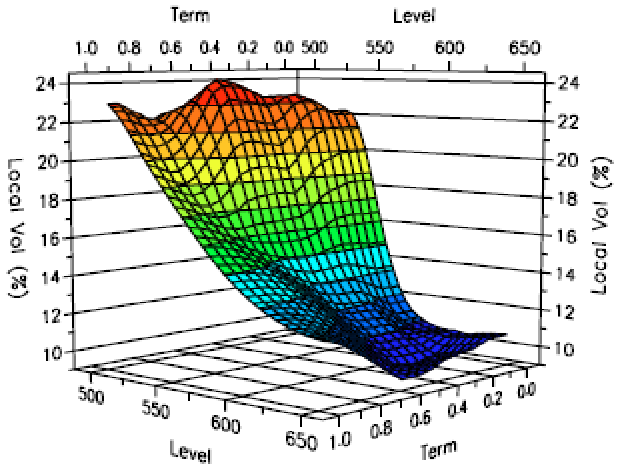

Figure (1): Implied volatility surface
Source: Derman, Kani, & Zou. (1996).

At the same time, Derman and Kani (1994) were also working on a similar model which was a perfect fit to volatility surface. Bates (1996) introduced a jump diffusion stochastic volatility model which used the best aspects of Heston's (1993) and Merton's (1976) model. The profound finding lead to a new area in research. Comte and Renault (1998) introduced the rough volatility model which used fractional Brownian motion to measure long term persistence memory in financial markets. Barndorff-Nielsen and Shephard, (2002) investigated realised volatility using high frequency data. A major drawback and criticism of the prior literature on stochastic models listed above is the fact that volatility was assumed to be constant rather than arbitrary. To this end, Christoffersen, Heston and Jacobs (2009) introduced the double Heston model which involved modelling two variance processes. One of the processes actually turned the correlation between return and volatility stochastic which was a major breakthrough in advancing the modelling financial markets volatility. Since then, there has been several studies on stochastic volatility models and option pricing. One of such studies was published by Osu et al. (2020): it investigated the impact of stochastic volatility on asset pricing and found that that different trajectory in stochastic parameters produced uncertainty in price history. Ventura (2021) investigated volatility smiles using stochastic Alpha and beta rho (SABR) model. The findings of Ventura's (2021) study indicates that short dated options have higher volatility which gradually decrease with the passage of time. Ke Peng, Xun & Hu (2021) also demonstrated that volatility premium curve can be used as a better predictor of implied volatility. The above literature presents an enormous attempt to capture market swings in financial markets. However, none of the above studies investigated stochastic volatility in financial markets especially during periods of financial distress. Therefore, this study attempts to fill the gap in literature.

3. Methodology

The required data was mainly daily share prices that were sourced from Yahoo finance. This data site provides credible financial information. These daily prices were used to calculate daily returns which is the most vivid projection of market volatility (Enow, 2022). In congruence with the aim of this study, the sample time periods were from, 1 December 2007 to 30 June 2009 which was the 2007-2008 financial crisis and 1 January 2020 to 31 December 2021 capturing the Covid-19 pandemic. Five financial markets namely; the Johannesburg Stock Exchange (JSE), Nasdaq, the French Stock Market Index (CAC 40), the German blue chip companies (DAX) and the Nikkei Stock Average (Nikkei 225).

In investigating stochastic volatility in financial markets, this study used the Heston (1993) model. This model was deemed suitable because it accounts for variations in price movement relative to its volatilities which assumes a random process (Blanco, 1996). The crux of this model lies in the use of two distinct stochastic processes for the mean and variance which are used to capture volatility trends (Aıt-Sahalia & Kimmel, 2007). To investigate stochastic volatility, the Heston (1993) model assumes that volatility is arbitrary which is a key factor compared to other traditional models which assumes a constant volatility. The Heston (1993) is given by;

$$dS_t = rS_t dt + \sqrt{V_t} S_t dW_{1t}$$
$$dV_t = k(\theta - V_t) + \sigma\sqrt{V}_t dW_{2t}$$
$$dW_{1t} dW_{2t} = \rho dt$$

Where W_{1t} is Brownian motion of the financial market,
W_{2t} is the Brownian motion of the financial Market's variance.
ρ is the correlation coefficient for W_{1t} and W_{2t}.
S_t is the market value of the financial market.
$\sqrt{V}_t$ is the volatility of the asset price.
σ is the volatility of the volatility or drift.
θ Long term price variance.
k is the rate of mean reversion to the long run price variance (Heston, 1993).

The main outputs in this model is the Variance mean reversion, correlation, drift and p-values. The section below presents the findings of the study.

4. Results and discussions

The tables below present the output results from the Heston (1993) model. Due to the vast amount of output, only the important findings are presented below.

2007-2008 Financial crisis results:

Table (1): Descriptive statistics for 2007 -2008 financial crisis

	JSE	*CAC-40*	*DAX*	*Nasdaq*	*Nikkei-225*
Mean	-0.101%	-0.112%	-0.088%	-0.054%	-0.083%
Standard Error	0.139%	0.121%	0.116%	0.127%	0.134%
Median	-0.025%	-0.078%	-0.008%	0.011%	-0.029%
Standard Deviation	2.718%	2.365%	2.280%	2.485%	2.626%
Sample Variance	0.074%	0.056%	0.052%	0.062%	0.069%
Kurtosis	2.61	4.07	4.88	3.46	4.21
Skewness	43.0%	47.6%	60.6%	27.2%	-2.8%
Range	21.6%	20.2%	18.6%	23.1%	25.6%
Minimum	-10.0%	-9.0%	-7.2%	-10.5%	-11.4%
Maximum	11.6%	11.2%	11.4%	12.6%	14.2%

Table (2): Correlation Matrix between financial Markets for 2007 -2008 financial crisis

	JSE	*CAC-40*	*DAX*	*Nasdaq*	*Nikkei-225*
JSE	1				
CAC-40	-0.045	1			
DAX	0.153	-0.046	1		
Nasdaq	0.156	-0.024	0.415	1	
Nikkei-225	0.006	-0.064	0.077	-0.022	1

Table (3): Heston model output for 2007-2008 crisis

	Long run Variance	*Variance mean reversion*	*Volatility of Variance*	*Correlation*	*Drift*	*Log likelihood*	*Likelihood ratio*	*p-value*
JSE	7.32	0.75	14.62	0.03	-0.13	-2536.47	3021.82	0.00
Nasdaq	51.90	1.00	943.83	-0.96	-0.39	-4079.02	987.22	0.00
CAC 40	5.42	0.83	12.86	0.00	-0.02	-2436.73	31.38	0.00
DAX	61.22	0.98	1135.43	-0.98	-0.42	-4068.31	1212.40	0.00
Nikkei 225	6.68	0.69	16.19	-0.31	0.01	-2539.85	105.41	0.00

Covid-19 pandemic results:

Table (4): Descriptive statistics for Covid-19 pandemic

	JSE	*CAC-40*	*DAX*	*Nasdaq*	*Nikkei-225*
Mean	0.02%	0.04%	0.04%	0.14%	0.05%
Standard Error	0.08%	0.07%	0.07%	0.08%	0.06%
Median	0.03%	0.10%	0.07%	0.29%	0.03%
Standard Deviation	1.78%	1.58%	1.61%	1.83%	1.42%
Sample Variance	0.03%	0.03%	0.03%	0.03%	0.02%
Kurtosis	2.47	12.00	12.84	8.51	4.33
Skewness	4.10%	-105.86%	-66.63%	-53.98%	22.53%
Range	15.03%	20.67%	23.21%	22.27%	14.12%
Minimum	-8.82%	-12.28%	-12.24%	-12.19%	-6.08%
Maximum	6.22%	8.39%	10.98%	10.07%	8.04%

Table (5): Correlation Matrix between financial Markets for Covid-19 Pandemic

	JSE	*CAC-40*	*DAX*	*Nasdaq*	*Nikkei-225*
JSE	1				
CAC-40	0.170	1			
DAX	0.220	0.574	1		
Nasdaq	0.128	0.250	0.346	1	
Nikkei-225	-0.077	0.085	0.014	0.058	1

Table (6): Heston model output for Coivd-19 Pandemic

	Long run Variance	*Variance mean reversion*	*Volatility of Variance*	*Correlation*	*Drift*	*Log likelihood*	*Likelihood ratio*	*p-value*
JSE	2.84	0.76	5.90	-0.27	-0.13	-1269.76	59.45	0.00
Nasdaq	3.73	0.48	9.36	-0.10	0.00	-2790.92	312.92	0.00
CAC 40	1.99	0.94	4.43	-0.01	0.09	-1203.77	11.77	0.07
DAX	25.30	0.99	364.27	-0.95	-0.36	-2357.07	621.91	0.00
Nikkei 225	2.14	0.54	4.32	-0.06	0.10	-2218.59	209.05	0.00

4.1 Stochastic volatility during the 2007-2008 Financial Crises

Tables 1 and 2 present the results of the descriptive statistics and correlation matrix for the 2007-2008 financial crisis. From table 1, the mean returns for all the financial markets under consideration are negative. As expected, these financial markets produced a negative return during the crisis. This finding is closely related to the skewness where all the values in Table 1 are greater than 2.5 indicating an asymmetric distribution. The returns of these financial markets also display an abnormal distribution where a score of at least 2.5 was obtained for the kurtosis. The data points are well above the mean from the values of the standard deviation in table 1. Moreover, Table 2 indicates weak positive and negative correlation between the financial markets which may be due to greater market efficiency.

The parameters from Tables 2 and 3 above presents some interesting findings of stochastics volatility during the 2007/2008 crisis and the pandemic. During the financial crisis in table 2, the correlation values of the Nasdaq, DAX and Nikkei are negative indicating that innovations of the mean and variance are inversely related. In other words, when there is a positive upward shock in the volatility, there will be a low return in the stock index. These positive shocks could represent flight to quality or macro-economic uncertainty. These findings correlated with concept that during uncertainty, markets participants tend to be risk adverse towards risky assets such as stocks and increase their holdings in bond portfolios. Accordingly, market participants and investors perceived the Nasdaq, DAX and Nikkei as risky investments. Conversely, the findings indicate a significant positive correlation in the JSE and CAC 40 during indicating that investors were optimistic about the level of risk and return in these markets. Also, the Nikkei 225 had a positive drift of 1% connoting an increase in value during the financial crisis while and the JSE, Nasdaq, CAC 40 and the DAX all had negative drifts ranging from 2% to 49%. Furthermore, the Nasdaq showed robust resilience during the financial crisis where it had a perfect mean reversion of 1 which is quite fast compared to the other financial markets where the mean reversion was 69% in the Nikkei, 75% in the JSE, 83% in the CAC 40 and 98% in the DAX. In essence, the variance in the Nasdaq, returns back instantly to its long run level while the long-run variance in the Nikkei only has a 69% recovery rate. The p-values for all the financial markets under consideration is statistically significant at 5% during the financial crisis indicating that the parameters are indeed reliable and the Heston model adds explanatory power to the model.

4.2 Stochastic volatility during the Covid-19 pandemic

The findings in Table 4 shows a slightly different outcome from that of the financial crisis. The mean return for all the financial markets under consideration was positive during the Covid-19 pandemic. The data points for all the financial markets were also well above the mean but slightly lower than that of the financial crisis. The kurtosis values also indicated high degrees of abnormality where the values are well above 2.4 as well as an asymmetric distribution to the right and to the left from the values of skewness. During the pandemic, all the financial markets under consideration were positively correlated to one another with the exception of the JSE and Nikkei-225 although the level of correlation was also weak. However, the findings in Table 3 for the Covid-19 pandemic presents a different picture where the correlation coefficients for all the financial markets under consideration where significantly negative except for the CAC 40. These findings confirm the vicious cycle of fear and loss of confidence in security markets around the world during the Covid-19 pandemic. Also, the CAC 40 and Nikkei 225 experienced a positive drift of 9% and 10% respectively while the JSE and DAX feel by 13% and 36% respectively. The variance mean reversion in the Nikkei 225 was 48% during the pandemic which was a massive drop from the financial crisis. This finding seems to infer that some form of market inefficiencies now exists in the Nasdaq. Finally, the mean reversion slightly improved during the financial crisis. Also, the Heston (1993) model parameters are significant at 5%.

5. Conclusion

The aim of this study was to explore stochastic volatility in financial markets during periods of financial distress namely; the financial crisis and Covid-19 pandemic. This study utilised a Heston model and a sample of five financial markets. The results indicate stochastic volatility and on average, a slower mean reversion during the financial crisis than the Covid-19 pandemic. Also, the findings contradict prior literature (Fouquau & Spieser, 2014; Lillo & Farmer, 2022) which contends that financial markets tend to have significant long run memories. The findings of this study also suggest that today's price charts and price actions will partly be a forecast of actionable trading levels in the future in most financial markets. We expect to see a shift from fundamental portfolio strategies to technical analysis due to effect of the Covid-19 pandemic on financial markets. Also, the findings of this study suggest mispricing in financial securities and deviation from fundamental values due the presence of stochastic volatility caused by the pandemic. Amalgamating the above mentioned finding means that there will be an increase in arbitrage opportunities in financial markets. Further studies should explore the extent to which these stochastic volatilities affect the fundamental values of asset pricing.

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